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A STATISTICAL STUDY OF VARIATION IN THE CATCH OF PLANKTON NETS[1]

By

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INTRODUCTION

Statement of the Problem.—In a previous communication, Winsor and Walford (1936) analyzed the results of repeated vertical hauls of different types of plankton nets, making use of data available in the literature. They concluded that the observed results could be accounted for reasonably well on the assumptions that (a) the organisms were randomly distributed in the medium and (b) that the volume of medium effectively sampled varied from haul to haul. From this result it follows that as the total catch is increased (as by increasing the net size) the reliability of a population estimate is ultimately limited by variation in volume sampled; and that where such conditions hold, it is useless to attempt greater accuracy by increasing either the total catch or the fraction counted much beyond the point where 100 organisms of the kind in question are counted.

The data available in the literature did not warrant further attempts at analysis. We have felt, however, that it should be possible to go further, and to determine to some extent the factors underlying the variability. We now present the results of a study of variation in successive and simultaneous hauls under various conditions.

The general problem of the reliability of results obtained by the use of plankton nets involves a number of different questions. In the present

[1] Woods Hole Oceanographic Institution Contribution No. 242.

paper we shall be primarily concerned with the variation attaching to the numbers caught; that is, within what limits may we expect repeated hauls in the same body of water to vary? An answer to this problem is evidently fundamental in any comparison of observations, and in answering such questions as the relative catching power of different nets.

It is evident, of course, that many other questions arise in evaluating the results of plankton hauls. For example, we have questions of the efficiency of nets, and how we are to pass from the numbers caught to the numbers present in the water. With such questions as these we are not here concerned.

We shall attempt here, then, to investigate the variability of repeated hauls in the same body of water, and the relation of this variability to size of net and manner of hauling.

Two other problems ought to be mentioned, though we shall not be concerned with them to any extent. First, there is the problem of laboratory sampling of the catch. This has been treated by others, and there seems to be no doubt that when the work is carefully done, variation from this source is comparatively small and follows well known probability laws. Second, how far in time and space is a particular catch to be taken as representative? We shall not discuss this problem, but some of our data will perhaps be found to have some bearing on it.

Material.—We have made use primarily of material of our own collection; we have also analyzed published data insofar as we have been able to use it. Our own collections consist of seven sets of hauls, made from the 40 foot power boat "Asterias" at locations within a day's run from Woods Hole. We are indebted to Mr. Dean F. Bumpus for technical assistance in the work at sea and in the subsequent enumeration of the catches. This material is more completely described below. In addition, we have made use of data published by Gardiner (1931) and Künne (1933).

Enumeration of Catch.—Each catch was diluted, mechanically stirred, and sampled as described by Clarke (1933). Those species which were present in adequate numbers were counted under the microscope. The following routine was adopted for the counting: for each species, counts were made in two samples; further samples, up to five, were counted until the total counted had exceeded 100. Occasionally, especially with the smaller nets, the entire catch of some species was counted.

Our tables give the estimated total catch for each species, and the number of samples counted in each case. The actual numbers counted can be obtained, if desired, by the use of the factors given in the tables.

The animals enumerated in our work were the following:

Calanus finmarchicus	Copepods
Metridia lucens	
Centropages typicus	
Centropages hamatus	
Pseudocalanus minutus	
Tortanus discaudatus	
Euthemisto sp. (juvenile)	Amphipod
Sagitta elegans (up to 22 mm.)	Chaetognath
Limacina retroversa	Pteropod

In several instances a species was enumerated by sex, or by development stages. We have therefore used the term group to indicate a category of animals enumerated separately.

Statistical Definitions.—We give definitions of certain statistical terms which we shall use:

The *variance* of a set of numbers is the sum of the squares of their deviations from their mean, divided by one less than their total number. That is:

$$v = \frac{S\,(x - \bar{x})^2}{n - 1}$$

The *standard deviation* of the set is the square root of the variance.

Note that we divide by $n - 1$ rather than by n.

A basic proposition in probability of which we shall make much use is the following:

Let x and y be independent random variables with mean values zero; let the mean values of their squares be σ_x^2 and σ_y^2; then the mean value of the square of their sum, σ^2_{x+y}, is

$$\sigma^2_{x+y} = \sigma_x^2 + \sigma_y^2$$

Analysis of Variance.—Given a set of data which can be divided into subsets, we can compare the variation within and between sets by means of the arithmetical procedure introduced by R. A. Fisher, known as the analysis of variance. Thus, if $x_1, x_2, \cdots x_n; y_1, y_2, \cdots y_n; z_1, z_2, \cdots z_n; \cdots$ are m sets of n observations, we can compare the variability of the means of the sets with the variability of the observations within each set. We can thus determine whether the means vary significantly more than would be expected from the within-set variability, and estimate the amount of variability which has been added to the means.

OBLIQUE HAULS

The problem with which we are here concerned is the estimation of the variability of plankton catches in a series of parallel hauls, all sampling

the same body of water. We are endeavoring to identify variations arising from different sources, and to determine the precision which attaches to a single observation made under conditions like those of our experimental series. We begin with the analysis of a series of oblique hauls, which will serve to illustrate our procedure in its simplest form. This series (Series I) consisted of twelve hauls, each of 10 minutes duration, made using two 75 cm. scrim nets (A and B) in succession according to the scheme AABBAABB—. The two nets used were of identical construction and dimensions. The original analysis was carried through keeping the nets distinct; but since no differences between them could be found (as was to be hoped), the distinction between them has been ignored in the analysis presented here.

The estimated catches are given in Table I, together with the mean and the range of variation for each group. It is apparent that the range of variation is very different for the different groups; and that this range is roughly proportional to the mean for each group. We can also observe a tendency for the numbers in any haul to be high or low together, indicating factors common to the whole haul affecting all groups in the same direction.

Under these circumstances, it is both convenient and theoretically advantageous to use the logarithm of the catches rather than the catches themselves. If we transform to logarithms we get the results found in Table II.

We now propose to assume that the factors determining the numbers in this table can be analyzed into the following:

(a) A mean for each group.

(b) A variation common to each count in a haul, differing independently from haul to haul.

(c) A residual variability, affecting each figure in the table independently.

We shall further assume that the variations under (b) and (c) are independent of each other and of the group mean, and we shall treat them as random deviations. We have thus set up a hypothetical scheme of the form,

$$y_{ij} = n_{i-} + h_{-j} + e_{ij}$$

where y_{ij} = log catch of i^{th} group in j^{th} haul
n_{i-} = true mean of i^{th} group
h_{-j} = haul deviation of j^{th} haul, common to all groups
e_{ij} = individual deviations of i^{th} group in j^{th} haul

It is now our purpose to estimate the variances (squared standard deviations) σ_H^2 and σ_{GH}^2 of the haul and individual deviations. For this purpose the computations can be very conveniently carried through in the form of Fisher's analysis of variance, giving the following table:

	Degrees of freedom	Sum of squares	Mean square
Total	47	7.4376	
Group	3	6.8298	
Hauls	11	.3500	.03182
Interaction, G × H	33	.2578	.00781

Now the mean square for interaction, .00781, is an estimate of the quantity σ_{GH}^2, the variance associated with the individual cells of the table. The mean square for hauls, .03182, is an estimate of the quantity $4\,\sigma_H^2 + \sigma_{GH}^2$, whence we get .00600 as an estimate of σ_H^2, the variance associated with the haul.

From the estimates .00781 and .00600 for the within-haul and haul to haul variances, we obtain .0884 and .0775 as the estimated standard deviations. These are obtained from the logarithms of the catches. To interpret these figures in terms of variability of the actual catches we proceed as follows:

$$.0884 = \log 1.226$$

$$.0775 = \log 1.195$$

Now a deviation of .0884 in the logarithm of the catch means that the catch has been multiplied (or divided) by 1.226. Hence we may say that one standard deviation in the logarithm corresponds to a percentage standard deviation, or coefficient of variation, of 22.6% in the catch. Similarly, a logarithmic standard deviation of .0775 corresponds to a coefficient of variation of 19.5%.

We can thus say: the complete suppression of haul to haul variation would leave variation such that each group would fluctuate about its own mean with a standard deviation of about 22.6%. The haul to haul variation itself, if all other variability could be eliminated, would be about 19.5%.

Actually, we are principally interested in knowing the uncertainty attaching to a single observation, because usually we deal with the results of single hauls. The variance of a single observation will be

$$\sigma^2 = \sigma_H^2 + \sigma_{GH}^2 = .01381$$

The standard deviation of a single observation will be the square root of this, or .117. This, we remember, is obtained from the logarithms of the catches; we have, therefore, as the percentage standard deviation, since

$$\log 1.31 = .117,$$

$$31\% = 131 - 100,$$

a value of 31%.

If we are interested in the difference between two catches, we shall usually wish to know the standard error of this difference. We shall have two cases: first, the comparison of two groups from a single haul, and

second, the comparison of catches from two different hauls. In the first case the error variance affecting the comparison is evidently $2\,\sigma_{GH}^2 = .01562$, giving a standard error for the difference of the logarithms of .125, or a percentage standard error for the ratio of 33%. For the difference of the logarithms of two catches from different hauls we have a variance of $2\,(\sigma_H^2 + \sigma_{GH}^2) = .02762$; the corresponding standard error is .166, and the percentage standard error of the ratio is 47%.

It may be well to point out that with percentage standard errors as large as those with which we are dealing, the usual concept of a standard error as giving equally probable plus and minus deviations may be misleading or even absurd. It is probably better to make any critical comparisons directly on the logarithms, and if numerical upper and lower limits, such as $\pm$ twice or three times the standard error, are desired, to find these for the logarithms, subsequently converting if necessary.

VERTICAL HAULS

(a) *Present Series.*

Our data on vertical hauls come from three sources, our own collection, and data published by Gardiner (1931) and by Künne (1933). Our data and Künne's were collected primarily for the purpose of comparing the catching power of nets, and the analysis of the results is somewhat complicated by this fact. We begin with our own data (Table III).

Here we have 8 hauls, 4 each with a ¾ meter and a 30.5 cm. net, hauled alternately. Six groups were counted, making a total of 48 observations. If now we set up the analysis of variance table, we have clearly 5 degrees of freedom from the 6 groups, and 1 from the two nets; there are then 5 degrees of freedom for net-group interaction, i. e., for differential catching power of the two nets. Considering now the haul to haul variations, we observe that the four pairs of hauls, one with each net, will give us three degrees of freedom representing differences between the totals of the pairs, and three degrees of freedom within the pairs, representing the failure of the totals for each net to agree with the overall total for each net. These two sets of three degrees of freedom may be expected to differ if there is any progressive change in population density with time; their agreement may be taken as indicating that all hauls have sampled the same population.

The remaining 30 degrees of freedom represent variations from haul to haul which affect the individual groups independently. They may be subdivided into a set of 15 between pairs of hauls, and a set of 15 within pairs, corresponding to the subdivision of the haul totals. Here again, agreement between these two sets will indicate that the composition of the population has remained constant throughout the series of hauls.

The analysis of variance corresponding to this subdivision follows:

	Degrees of freedom	Sum of squares	Mean square
Total	47	4.5392	
Group	5	2.3499	.4700
Net	1	.8560	.8560
Net × Group	5	.1002	.0200
Hauls between pairs	3	.2169	.0723
Within pairs	3	.3897	.1299
Group × Haul between pairs	15	.3887	.0259
Within pairs	15	.2378	.0158

From this table it appears that the mean squares between and within pairs do not differ significantly, either for the haul totals or for the Group × Haul interactions. We may therefore reasonably conclude that we have been sampling from an unchanging population throughout. Further, we note that the Net × Group mean square, .0200, does not differ significantly from the Group × Haul mean squares; and we therefore conclude that the groups here considered are caught in the same proportions by these two nets.

If we accept these conclusions, we may reasonably regroup the analysis of variance table as follows, putting the Net × Group together with the Group × Haul:

	Degrees of freedom	Sum of squares	Mean square
Total	47	4.5392	
Group	5	2.3499	.4700
Net	1	.8560	.8560
Haul	6	.6066	.1011
Group × Haul	35	.7267	.0208

If now we seek as before, to separate the within haul and haul to haul variances, σ_{GH}^2 and σ_H^2, we have:

$$.1011 = 6\,\sigma_H^2 + \sigma_{GH}^2$$
$$.0208 = \sigma_{GH}^2$$

whence we get

$$\sigma_H^2 = .0134,\ \sigma_H = .116$$
$$\sigma_{GH} = .144$$

As the standard deviation of the logarithm of a single observation we have $\sqrt{.0134 + .0208} = .185$, which corresponds to a percentage standard deviation of about 53%.

It may be interesting to compare these figures with those previously obtained for oblique hauls.

	Vertical	Oblique
σ_{GH}	.144	.0884
σ_H	.116	.0775
$\sigma = \sqrt{\sigma_H^2 + \sigma_{GH}^2}$	.185	.117
s.d. as % of mean	53%	31%

In this particular set of data the oblique hauls have been considerably more consistent than the vertical hauls. We do not assert that this will always be true; but it suggests to us that the accuracy of oblique hauls may well be worth further investigation.

A possible offset to the apparent advantage of the oblique haul is the extra time required. If we argued that two vertical hauls could be taken and counted in the time for one oblique haul, we should halve the variance of a single observation, giving a standard deviation of about .130, or a percentage standard deviation of about 35%.

We may now inquire: what is the ratio of the catches of the 75 cm. and 30.5 cm. nets used in this series, and how precisely has this ratio been determined? The mean of the differences of the logarithms of the 30.5 cm. and 75 cm. catches is $\bar{1}.086$, corresponding to an actual ratio of the catches of 0.122. The variance of the logarithm of this ratio is $2 \times .1011/24 = .00842$, giving a standard error for the logarithm of 0.092. (We use .1011, the mean square for hauls, rather than .0208, because haul to haul variation affects our ratio.) We may now argue: it is very improbable that a subsequent comparison would give a value differing from the observed $\bar{1}.086$ by more than three times the standard error; or, it is practically certain that the logarithm of the ratio will lie between $\bar{2}.810$ and $\bar{1}.362$; or, finally, the ratio itself is practically certain to lie between .065 and .230.

The ratio which would be expected from the openings of the nets is .165. This is larger than the observed .122; but the difference is not certainly established. We may remark, however, that our other material shows a similar tendency for smaller nets to catch less than would be expected from their openings.

As against this, the two nets appear to catch all the groups here discussed in the same proportion. There is no evidence that either net is relatively favorable or unfavorable to any particular group. This, it may be added, agrees with the results obtained by Künne (1933) on larger nets.

Our second series of vertical hauls consists of 30 hauls, using a 75 cm. net, a 30.5 cm. net, and a 12.7 cm. net in rotation. The data are given in Table IV. Unfortunately, only one species (*Centropages typicus*) was present in sufficient numbers in all three nets to warrant critical comparison. *Sagitta elegans* was also present in the hauls, but in numbers so small, in the two smaller nets, as to make doubtful the validity of comparisons by our previous methods. We have, however, made the computations for both species in the two larger nets.

Since the three nets were hauled independently, we have analyzed them separately, with the following results:

	Mean Squares		
	75 cm.	*30.5 cm.*	*12.7 cm.*
Haul	.3140	.1512	
G × *H*	.0244	.0570	.0346

Note that since we have only one species in the smallest net, the breakdown into haul and G × H is not possible.

It will be observed that the 75 cm. net shows a significantly greater mean square for haul than for G × H. For the 30.5 cm. net the difference is not statistically established, but is in the same direction. Examination of the data suggests that there is a difference between the *Sagitta* and *Centropages* figures. If we compute the mean squares by species, we get the following:

	75 cm.	*30.5 cm.*	*12.7 cm.*
Centropages	.1705	.0328	.0346
Sagitta	.1680	.1754	

Here in the 30.5 cm. net the mean square for *Sagitta* is significantly greater than that for *Centropages*. To judge by the catch figures, there was apparently a change in the population, the early hauls catching more *Sagitta* than the later.

The ratio of catches of the three nets, taking the 75 cm. net as 1, is:

	75 cm.	*30.5 cm.*	*12.7 cm.*
Centropages	1	.165	.0315
Sagitta	1	.132	
Both species	1	.148	

It remains to be inquired whether the 12.7 cm. net has caught *Sagitta* in the proportions which would be indicated by the catch ratio for *Centropages*. We may reach an estimate of the total catch of *Sagitta* to be expected in the 12.7 cm. net in the ten hauls as follows:

In the first haul 241 *Centropages* were taken; with the 75 cm. net, 14,750 *Centropages* and 123 *Sagitta* were taken; hence, we would expect $\frac{241 \times 123}{14750}$ = 2.0 *Sagitta*. Proceeding for the succeeding hauls, we get an estimated catch in all ten hauls of 20.9 *Sagittae*, against an observed total of 9. Testing this by χ^2 with one degree of freedom, we find a probability of somewhat less than .001 that such a difference could have arisen by chance. It appears highly probable, then, that in this series the smallest net has been less efficient in catching *Sagittae* than in catching *Centropages*. The data, however, are not sufficiently extensive to allow us to go farther than this.

(b) *Gardiner's Data*

Gardiner (1931) has published data on vertical hauls made with the international net. We have used here his second series. Gardiner lists

three groups of animals and the total catch. We have used his three groups (*Calanus, Temora,* and *Pseudo-* and *Paracalanus*) and a fourth (all others) obtained by subtraction.

Inspection of his data suggests that during the hauls (which extended over some eleven hours) there have been progressive changes in the population, which it would be desirable to eliminate. We have therefore paired the hauls (so far as this was possible), pairing hauls taken within 10 minutes, and determined the variation both within and between pairs. The analysis of variance table runs:

	Degrees of freedom	*Sum of squares*	*Mean square*
Total	143	13.7486	
Groups	3	6.8551	
Hauls	35	5.0202	
Between Pairs	20	4.7113	.2356[a]
Within Pairs	15	.3089	.0206[b]
Groups × *Hauls*	105	1.8733	
G × *between*	60	1.2933	.0216[c]
G × *within*	45	.5800	.0129

[a] Significant at 1% level.
[b] .0206/.0129 = 1.59; F(5%) = 1.89, d.f. 15 and 45.
[c] .0216/.0129 = 1.67; F(5%) = 1.59, d.f. 60 and 45.

From this table it is clear (a) that there is significant variation in the totals from one pair of hauls to another; but (b and c) it is not certainly established either that there is variation in the haul totals within pairs, or that there is variation in the composition of the population from pair to pair. We may, however, reasonably observe that the changes to be expected in the population are gradual, and we may ask whether grouping hauls taken within an hour would lead to different conclusions. We have then:

	D.F.	*Sum of squares*	*Mean square*
Total	143	13.7486	
Groups	3	6.8551	
Hauls	35	5.0202	
Between hours	7	3.6346	.5192[a]
Within hours	28	1.3856	.0495[b]
Groups × *Hauls*	105	1.8733	
G × *between*	21	.5922	.0282[c]
G × *within*	84	1.2811	.0152

[a] Significant at 1% level against .0495.
[b] Significant at 1% level against .0152.
[c] .0282/.0152 = 1.85; F(5%) = 1.69; F(1%) = 2.07.

Comparing this table with the preceding, we see that variation of haul totals within hours is significant; and that the variation of group proportions from hour to hour is probably significant.

If now we attempt to carry through our preceding analysis to determine the variances associated with the different categories, we have the difficulty that the number of hauls is not the same for each hour. We can, however, get the following estimates:

σ_H^2 : .0265 haul to haul variance totals between hrs.
σ_{HW}^2 : .0086 haul to haul variance totals within hrs.
σ_{GH}^2 : .0030 variance between hrs. species to species.
σ_{GHW}^2: .0152 species to species variance within hrs.

These estimates may now be used to give us the reliability of a single observation. If our observation is to be regarded as representative of the hour during which it was made, its variance will be $\sigma_{HW}^2 + \sigma_{GHW}^2$, or .0238. From this we get a standard deviation of .154, or a percentage standard deviation of 43%. If we desired to take the haul as representative of a longer period, as of the whole twelve hours, we would have a variance of .0533, or a standard deviation of .231, giving a percentage standard deviation of 70%.

Künne's Data

Künne's data afford the most extensive series of net comparisons with which we are acquainted. Unfortunately for our present purposes, not many of these hauls contain large catches of several species. We have chosen for analysis his series 62–65 and 73–76, all from his Table 8, in which the Helgoland larva net is compared with the Hensen egg net. In these hauls, a complete series consisted of two hauls of the Helgoland larva net, preceded and followed by a haul of the Hensen net. The analysis of variance runs:

	Series 62–65			*Series 73–76*		
	D.F.	*S. Sq.*	*Mean Sq.*	*D.F.*	*S. Sq.*	*Mean Sq.*
Total	79	43.4623		63	10.4320	
Groups	4	33.0675	8.4169	3	3.0573	1.0191
Net	1	7.7626	7.7626	1	5.8504	5.8504
$N \times G$	4	.0458	.0114	3	.0027	.0009
Haul totals						
Between pairs	7	.2766	.0395	7	.0432	.0062
Within pairs	7	.2943	.0420	7	.1976	.0282
$G \times$ *Between*	28	.6127	.0219	21	1.0356	.0493
$G \times$ *Within*	28	.8028	.0287	21	.2452	.0117
	70	1.9864		56	1.5216	

We may now remark that there is no significant difference between the mean square of haul totals for between and within pairs; nor, in the case of series 62–65, is there any difference in the interaction G × Between and G × Within. In series 73–76 there is a significantly greater mean square (.0493 vs. .0117) for G × Between than for G × Within, indicating significant changes in the percentage composition of the population during the series of hauls. On the whole, however, it seems reasonable to combine the G × Between and G × Within interactions. Further, there seems to be no indication of any essential difference between the two sets of data as regards haul mean square or G × Haul interaction. We may therefore reasonably compute a mean square for error by combining the last four lines from both tables, giving:

D.F.	*S. Sq.*	*Mean Sq.*	*S.D. of log.*	*S.D., % mean*
126	3.5080	.0278	.167	47%

This is a somewhat greater variability than Winsor and Walford found for Künne's data as a whole. They indicated that a coefficient of variation of around 25% seemed to give a good fit to Künne's data. The discrepancy does not seem serious to us, for several reasons. First, we are dealing here with only a small portion of Künne's data. Second, Winsor and Walford were dealing entirely with discrepancy between pairs of catches; as we have said, there is evidence in the analysis above of variation in the composition of the population during the entire series of hauls. We should therefore expect our computations to show a greater variation.

We may conveniently summarize our work on the precision of vertical hauls with a comparison of standard deviations, expressed as the standard deviation of the logarithm of a single observation, and also as a percentage of the mean.

	σ of log.	*σ, % of mean*
Our data	.185	53%
Gardiner's data	.154	43%
Künne's data	.167	47%

The agreement between these standard deviations seems to us very satisfactory. We conclude that in vertical hauls made with good technique we may expect that the standard deviation of a single observation will be from 45 to 50% of the mean.

HORIZONTAL HAULS

(a) *Statistical Analysis.*

We are not aware of any published study of the variability of horizontal hauls. We have data on four series of such hauls, two of which are extensive enough to warrant detailed analysis. The data are given in Tables VII,

VIII, IX and X. In the first two of these series three nets, a 75 cm. net and two 12.7 cm. nets were towed simultaneously. The large net was placed at 30 meters, and the small nets at one meter above and below. The bridles were adjusted to bring the mouths of the nets in the same vertical plane. The two small nets were alternated in the top and bottom position in successive hauls. In the third series, the two 12.7 cm. nets were towed simultaneously at 29 and 30 meters, without the large net.

We originally carried through the analysis keeping the two small nets separate. As was to be hoped, there was no evidence of difference between them; and they have therefore not been distinguished in the analysis presented here. As against this, however, marked differences exist between upper and lower nets, corresponding to considerable (apparent) vertical gradients in plankton density. These differences will be discussed at greater length below.

The analysis of variance for these hauls runs as follows:

Series V

	Sum of squares	*Degrees of freedom*	*Mean square*
Total	54.5735	149	
Group	36.1323	4	
Hauls	3.9880	9	.4431
$G \times H$	6.4360	36	.1788
Nets	5.6839	1	
$N \times G$	.3147	4	.0787
$N \times H$	.2708	9	.0301
$N \times G \times H$	.2495	36	.0069
Position	.3709	1	.3709
$P \times G$	.1572	4	.0393
$P \times H$	.6528	9	.0725
$P \times G \times H$	.3354	36	.0093

Series VI

	Sum of squares	*Degrees of freedom*	*Mean square*
Total	47.8319	143	
Group	29.2877	5	
Hauls	2.4040	7	.3434
$G \times H$	7.5384	35	.2154
Nets	3.7151	1	
$N \times G$	.1855	5	.0371
$N \times H$	.6606	7	.0944
$N \times G \times H$	.4508	35	.0128
Position	.1692	1	.1692
$P \times G$	2.0390	5	.4078
$P \times H$	.4981	7	.0712
$P \times G \times H$	.8835	35	.0252

Inspection of these figures shows that the corresponding mean squares in the two tables are of generally the same order of magnitude; the conspicuous exceptions are the mean squares for Position and Position × Group, which we shall discuss later. Aside from these, the two sets of figures are sufficiently nearly alike to indicate that essentially the same sources of variation have acted in both cases. We therefore have proceeded to combine the two sets to get estimates of these sources of variation.

We begin with the triple interactions, $N \times G \times H$ and $P \times G \times H$. Since in each series the triple interactions are essentially alike, it seems reasonable to suppose that they represent essentially the same residual variance. The two series, however, are based on different numbers of degrees of freedom; we therefore add the sums of squares and the degrees of freedom to reach a weighted final estimate of the residual variance, σ_{NGH}^2:

.2495	36	
.3354	36	
.4508	35	
.8835	35	
1.9192	142	$\sigma_{NGH}^2 = .01351$

We now consider the two factor interactions $N \times H$ and $P \times H$. These are of about the same order of magnitude. It seems reasonable to suppose that they reflect essentially variations due to the failure of nets attached to the same wire to fish exactly alike in successive hauls. Such variations might arise, for example, if the nets were not hung on the wire in exactly the same manner in each haul. If we call the component of variance arising from this source σ_{NH}^2, we have to observe that the mean squares for $N \times H$ and $P \times H$ in Series V are estimates of $5\,\sigma_{NH}^2 + \sigma_{NGH}^2$, while those in Series VI are estimates of $6\,\sigma_{NH}^2 + \sigma_{NGH}^2$; and that the degrees of freedom differ in the two cases. Proceeding as before, we add the sums of squares and degrees of freedom:

	Sum of squares	*D.F.*	*Quantity estimated by S. Sq.*
Series V $N \times H$.....	.2708	9	$45\,\sigma_{NH}^2 + 9\,\sigma_{NGH}^2$
$P \times H$.....	.6528	9	$45\,\sigma_{NH}^2 + 9\,\sigma_{NGH}^2$
Series VI $N \times H$.....	.6606	7	$42\,\sigma_{NH}^2 + 7\,\sigma_{NGH}^2$
$P \times H$.....	.4981	7	$42\,\sigma_{NH}^2 + 7\,\sigma_{NGH}^2$
	2.0823	32	$174\,\sigma_{NH}^2 + 32\,\sigma_{NGH}^2$

Deducting $32\,\sigma_{NGH}^2 = .4323$, derived from our previous estimate of σ_{NGH}^2, and dividing by 174, we get as an estimate of σ_{NH}^2, .0095.

We consider next the $N \times G$ interaction. This measures the extent to which the proportions of the different groups vary with net size—i. e., the amount of differential selection by nets of different sizes. This interaction

is statistically significant, that is, it represents more than mere random variation of the kind represented by the triple interactions; but its physical significance is not so clear. It may actually represent differential selection with net size; but it could also arise out of differential density gradients between groups. In either case we have to face the uncertainty introduced by this factor. If we regard the particular groups here counted as simply a random sample of the kind of organisms for which these nets are suitable, then a portion of our uncertainty in the interpretation of any count is attributable to what we must regard as random variation in differential catching power. Likewise, any part of the $N \times G$ interaction which really arises out of density gradients would seem appropriately to be regarded as part of the random disturbing elements.

As against this, if we proposed to confine our further investigations to the particular groups here studied, it would be appropriate to determine net ratios for each group separately, and use these ratios in our comparisons. This, however, is not the purpose of our present investigation.

Proceeding as in the case of the $N \times H$ interaction, we have:

	S. Sq.	*D.F.*	*Quantity Estimated*
324 $N \times G$	.3147	4	$40\,\sigma_{NG}^2 + 4\,\sigma_{NGH}^2$
327 $N \times G$	.1855	5	$40\,\sigma_{NG}^2 + 5\,\sigma_{NGH}^2$
	.5002	9	$80\,\sigma_{NG}^2 + 9\,\sigma_{NGH}^2$
$9\,\sigma^2 =$	.1216		
	.3786	$= 80\,\sigma_{NG}^2$	

$\sigma_{NG}^2 = .0047$

We may now consider the position effect and the $P \times G$ interaction. As has been said, we have compared the catch of the upper against the lower net. The mean square for position is a measure of the consistency with which the difference between these two is maintained over all groups. The $P \times G$ interaction measures the variation of this difference from group to group. It is apparent that in hauls 324, the density gradient was in the same direction for all groups; in hauls 327, there were differences in direction of the gradient. The actual values of the ratio upper to lower net for the two series were:

	Series V	*Series VI*
Calanus V	1.26	1.52
Calanus ♀	1.21	1.54
Metridia	1.27	0.28
Centropages	1.11	1.17
Euthemisto	1.83	1.67
Sagitta	—	1.93

The interpretation to be put on these figures is not entirely clear to us. Their statistical significance is well enough established, that is, we cannot reasonably suppose them to have arisen through random sampling errors. We hesitate, however, to assert that they give a true picture of the distribution in depth of plankton organisms. Were this the case, we should have to reckon with vertical distributions of population in which the density might change by two to three fold in a two meter range in depth. Such a variation is physically possible; but that the stratification should be so sharp and maintained so uniformly over a period of several hours, does seem to us so surprising that other possible explanations ought to be sought and examined.

One possible explanation (as to which, unfortunately, we have no evidence one way or the other) might lie in the reactions ("flight reactions") of the organisms to disturbance. If organisms characteristically reacted by an upward or downward movement to a disturbance, the result would be an apparent density gradient upward or downward. Such an explanation cannot be tested with our present knowledge. We hope to investigate this question in the future.

If now we ask: what information have we as to the effect of this density gradient effect, to whatever cause we attribute it, on the accuracy of our estimate of a particular group, we may reason as follows. If we propose to generalize from the particular groups caught in the present hauls, we are in effect assuming that there is a random component of density gradient whose magnitude we have to estimate from our data on Position and $P \times G$. We have, then, no reason for wishing to evaluate a variance for position as such, but rather we assume that there is a variance σ_{PG}^2 which enters into both the observed position and $P \times G$ mean squares. But we further observe that the position mean square differs from the $P \times G$ mean square by containing a part due to the $P \times H$ interaction. Actually the mean squares for position and for $P \times G$ are estimates of:

$$\text{Position: } n_H\sigma_{PG}^2 + n_G\sigma_{PH}^2 + \sigma_{PGH}^2$$

$$P \times G \quad n_H\sigma_{PG}^2 \qquad\qquad + \sigma_{PGH}^2$$

where n_H and n_G are the number of hauls and groups. Further, our two sets of hauls are based on different numbers of degrees of freedom. If we add the sums of squares for position and $P \times G$ from the two sets of data, we obtain:

	S. Sq.		Quantity Estimated
P	.3709		$10\,\sigma_{PG}^2 + 5\,\sigma_{PH}^2 + \sigma_{PGH}^2$
$P \times G$	.1572		$40\,\sigma_{PG}^2 + 4\,\sigma_{PGH}^2$
P	.1692		$8\,\sigma_{PG}^2 + 6\,\sigma_{PH}^2 + \sigma_{PGH}^2$
$P \times G$	2.0390		$40\,\sigma_{PG}^2 + 5\,\sigma_{PGH}^2$
Total	2.7363		$98\,\sigma_{PG}^2 + 11\,\sigma_{PH}^2 + 11\,\sigma_{PGH}^2$
	.2530	=	$11\,\sigma_{PH}^2 + 11\,\sigma_{PGH}^2$
	2.4833	=	$98\,\sigma_{PG}^2$
	.0254	=	σ_{PG}^2

(Note: σ_{PH}^2 and σ_{PGH}^2 have previously been estimated as .0095 and .0135 respectively.)

We next consider the question of the $G \times H$ interaction. This arises from the failure of all groups to be represented in the same proportions in the totals of each haul. To estimate the variance σ_{GH}^2 attributable to this variation we proceed as previously:

$$\begin{aligned} 6.4360 &= 108\,\sigma_{GH}^2 + 36\,\sigma_{NGH}^2 \\ 7.5384 &= 105\,\sigma_{GH}^2 + 35\,\sigma_{NGH}^2 \\ \hline 13.9744 &= 213\,\sigma_{GH}^2 + 71\,\sigma_{NGH}^2 \end{aligned}$$

whence we get

$$.0611 = \sigma_{GH}^2$$

Finally we have to consider the question of variation of the haul totals. It is not certain from the analysis of variance tables that there is a significant variation of the haul totals; but there seems to be reason to suppose that such a component of variation ought to exist (since it is hardly reasonable to suppose that speed, duration of haul, etc., are really devoid of variation from haul to haul). Further, in both series the mean square for hauls is actually greater than the $G \times H$ mean square. Proceeding, then, to determine the variance σ_H^2, we have:

$$\begin{aligned} 3.9880 &= 135\,\sigma_H^2 + 27\,\sigma_{GH}^2 + 45\,\sigma_{NH}^2 + 9\,\sigma_{NGH}^2 \\ 2.4040 &= 126\,\sigma_H^2 + 21\,\sigma_{GH}^2 + 42\,\sigma_{NH}^2 + 7\,\sigma_{NGH}^2 \\ \hline 6.3920 &= 261\,\sigma_H^2 + 48\,\sigma_{GH}^2 + 87\,\sigma_{NH}^2 + 16\,\sigma_{NGH}^2 \end{aligned}$$

Using the previous estimates of σ_{GH}^2, etc., we get .0093 as an estimate of σ_H^2.

We may summarize these results as follows:

$$\begin{aligned} \sigma_{NGH}^2 &= .0135 \\ \sigma_{NH}^2 &= .0095 \\ \sigma_{NG}^2 &= .0047 \\ \sigma_{PG}^2 &= .0254 \\ \sigma_{GH}^2 &= .0611 \\ \sigma_H^2 &= .0093 \end{aligned}$$

As before, we may ask: what is the standard deviation of a single observation made under conditions similar to those of the present investigation? That is, suppose we hauled a single net once, and enumerated a particular group; what estimate can we make of the precision of such a figure? If we simply add the variances just found, we get as the variance of a single observation .1235; the corresponding standard deviation is .351, giving a percentage standard deviation of 124%. It is probably more instructive to consider how the actual catch varies. Let us suppose that the actual catch is 1000 organisms; then the logarithm of the catch will be 3.000. For various multiples of the standard deviation in excess or defect of this we have the following:

Deviations	*log catch*	*actual catch*
-2σ	2.298	199
$-\sigma$	2.649	446
σ	3.000	1000
$+\sigma$	3.351	2240
$+2\sigma$	3.702	5040

This variability is evidently large. It may, however, reasonably be urged that the present series of hauls have extended over a considerable period of time—four of five hours—and it is quite possible that changes in the population may have occurred. This idea is supported by the large size of σ_{GH}^2, which reflects variations in the proportions of species in the different hauls. One may, then, desire to consider a shorter period of time than that covered by the whole of these hauls, say the time from one haul to another. In this case we shall group the hauls by pairs, and consider the variability within and between pairs. Carrying through the computation, we get as an estimate of σ_{GH}^2 within pairs .0199. Using this in place of the previous .0611, we get for the standard deviation of the logarithm of a single observation .286, or a percentage standard deviation of about 93%. Our previous table now becomes:

Deviations	*log catch*	*actual catch*
-2σ	2.428	268
$-\sigma$	2.714	518
σ	3.000	1000
$+\sigma$	3.286	1930
$+2\sigma$	3.572	3730

As a check on these computations, it is of some interest to examine the results obtained if we consider only the figures given by the large net. In this case we are able to use ten hauls in each series. The analysis of variance runs:

Hauls 324

	D.F.	S. Sq.	Mean Sq.	Estimate of
Total	49	8.1135		
G	4	4.6144	1.1536	
H	9	1.3411	.1490	$5\,\sigma_H^2 + \sigma_{GH}^2$
$G \times H$	36	2.1580	.0599	σ_{GH}^2

Hauls 327

Total	59	15.0179		
G	5	8.9180	1.7836	
H	9	1.7330	.1926	$6\,\sigma_H^2 + \sigma_{GH}^2$
$G \times H$	45	4.3669	.0970	σ_{GH}^2

The two estimates of σ_{GH}^2 do not differ significantly. We may therefore resonably combine the two sets of data to get an estimate of σ_H^2.

1.3411 est. of	$45\,\sigma_H^2 + 9\,\sigma_{GH}^2$
1.7330 est. of	$54\,\sigma_H^2 + 9\,\sigma_{GH}^2$
3.0741 est. of	$99\,\sigma_H^2 + 18\,\sigma_{GH}^2$
2.1580 est. of	$36\,\sigma_{GH}^2$
4.3669 est. of	$45\,\sigma_{GH}^2$
6.5249 est. of	$81\,\sigma_{GH}^2$
1.4500 est. of	$18\,\sigma_{GH}^2$
.0806 est. of	σ_{GH}^2

Whence we have 1.6241 as an estimate of $90\,\sigma_H^2$, and .0164 as our estimate of σ_H^2. As the variance of a single observation we shall then have .0164 + .0806 = .0970. This is lower than the .1235 we obtained previously; but we may observe that the previous estimate includes the (apparent) vertical density gradients as a component of error. We have previously suggested the possibility that these gradients are of a different nature from the usual error components. If we deduct from .1235 the component .0254 for σ_{PG}^2, we are left with .0981, which is very close to the figure .0970 for the large net alone. We do not lay particular stress on this agreement; it suggests, however, that the data as a whole are reasonably homogeneous.

(b) *Catching power of different sizes of nets.*

The determination of the relative catching power of the two nets is of interest, and it is also a matter of importance to know the precision of this determination. We may reason as follows:

In any particular haul, we have determinations of the catch on a number of groups. Letting *M, L, U* be the logarithms of the catch of the middle, lower and upper nets, the logarithm of the ratio is $\frac{1}{2}(2M - L - U)$. If

we have say, 10 hauls with five groups in each, we have a total of 50 determinations of the ratio. Our best estimate will be given by the mean of these 50.

Now to what errors is this estimate subject? We have suggested that the following sources of error should be considered as affecting our data:

$$\left.\begin{matrix}\sigma_H^2 \\ \sigma_{GH}^2 \\ \sigma_{PG}^2\end{matrix}\right\} \qquad \left.\begin{matrix}\sigma_{HN}^2 \\ \sigma_{NG}^2 \\ \sigma_{NGH}^2\end{matrix}\right\}$$

The first group does not affect the precision of our net ratio; the second group does. But we have also to observe that the second group contains three sources of error which act differently. σ_{HN}^2 represents error affecting all hauls on a particular species. σ_{NHG}^2 represents error varying independently from observation to observation.

Consider any single determination,

$$r = \tfrac{1}{2}(2M - L - U)$$

Let σ^2 be the variance of M, L, U. Then we have

$$\sigma_r^2 = \frac{6\,\sigma^2}{4}$$

But actually our r is the mean of 50 single values:

$$\bar{r} = \frac{1}{50} Sr;$$

and the errors in these 50 values are not independent. We have, actually, 10 hauls and 5 species. Instead, then, of having

$$\sigma_{\bar{r}}^2 = \frac{1}{50}\sigma_r^2 = \frac{1}{50}\cdot\frac{3}{2}\sigma^2$$

we shall have:

$$\sigma_{\bar{r}}^2 = \frac{1}{50}\cdot\frac{3}{2}(\sigma_{NGH}^2 + 10\,\sigma_{NG}^2 + 5\,\sigma_{NH}^2)$$

In our second series, we have 6 groups and 8 hauls; we get then

$$\sigma_{\bar{r}}^2 = \frac{1}{48}\cdot\frac{3}{2}(\sigma_{NGH}^2 + 8\,\sigma_{NG}^2 + 6\,\sigma_{NH}^2)$$

Substituting our estimates of the σ^2, we get

$$\sigma_{\bar{r}}^2 = \frac{1}{50}\cdot\frac{3}{2}(.1081) \text{ and } \sigma_{\bar{r}}^2 = \frac{1}{48}\cdot\frac{3}{2}(.1080)$$

As the variance of our combined estimate we shall have very closely

$$\sigma_r^2 = \frac{.3243}{196} = .001655$$

giving $\sigma_r = .0407$ as the standard error of the logarithm of the ratio of the 12.7 cm. net to the 75 cm. net.

The logarithm of the ratio in series 324 was 1.7122, and in series 327 was 1.6414. The (weighted) mean of these is 1.6775, and the standard deviation of this determination is .0407. Expressed as the ratio of catches of 12.7 cm./75 cm., we have a value .0210, and taking twice the standard error, we have limits .0174 and .0253 as lower and upper bounds within which the ratio is nearly certain to lie. The ratio calculated from the openings is .0285, which is well outside our limits. We may summarize by saying that the small nets apparently catch about 25% less than would be expected from the area of their openings relative to the 75 cm. net.

The question of differential selection by the nets deserves further consideration. We have previously indicated that the statistical significance of the net × group interaction is sufficiently established, but we also pointed out that its interpretation was not completely clear. We may tabulate the net ratios by groups for the two series separately.

	Series V	*Series VI*
Calanus V	.0152	.0211
Calanus ♀	.0163	.0243
Centropages	.0214	.0256
Metridia	.0193	.0283
Euthemisto	.0286	.0232
Sagitta	—	.0163
All groups	.0198	.0228

If there were any important differential selection by the nets, we should expect that the same groups would show high or low ratios in both series. No such consistency is apparent, and we therefore incline to the belief that the differences in net ratio for different species do not reflect true differences in the nets, but rather random effects which are properly to be included in observational error.

Since the variation of the smallest net is no greater than that of the nets with which it was compared and since there is no indication that the small net fails to catch large and small, active and inactive animals in the same proportion as the large net, we may conclude that under the present conditions of work a net as small as 12.7 cm. in diameter is as reliable for the groups of plankton considered as a net of the more usual size (75 cm. in diameter). Small nets are cheaper, are more easily handled in rough weather, can be more readily rigged for closing, and may be furnished with a

volume meter. A device for the quantitative investigation of zooplankton which incorporates these advantages has recently been developed in the form of the "plankton sampler"—a 5-inch (12.7 cm.) tube and net equipped with shutter and meter (Clarke and Bumpus, 1939).

In addition to the two series of horizontal hauls just discussed, we have a short series (VII) of horizontal hauls made with the two small nets. The data are presented in Table IX. Here inspection shows that there is no such clear evidence of density gradients as in the preceding series. If we carry through the analysis of variance on the logarithms, omitting Sagitta, because the numbers are small, we obtain:

	D.F.	*S. Sq.*	*Mean square*
Total	49	12.6197	
Groups	4	7.8496	
Hauls	4	.2611	.0653
$G \times H$	16	3.8027	.2377
Position	1	.0188	.0188
$P \times G$	4	.0569	.0142
$P \times H$	4	.2777	.0694
$P \times G \times H$	16	.3528	.0220

It is clear that there is no evidence here of any density gradients. This fact suggests that the presence of the 75 cm. net was responsible in some way for the gradient observed in the other series. The mean square for $P \times G \times H$ agrees well with the figure previously obtained of .0135. The $G \times H$ mean square is large; this would be expected from examination of Table IX, since this table shows considerable changes with time in the catch of several of the groups. A breakdown into between and within pairs of hauls, pairing the first two and last two hauls, gives a mean square for $G \times H$ within hauls of .0317, which is not far from the figure previously found. It appears, therefore, that except as regards density gradients, the present series agrees well with those previously treated.

The absence of any significant density gradients in this series suggests, of course, that the gradients previously found are in the nature of artifacts. This is the more true because this series was made immediately following our Series VI. We can only repeat our previous suggestion that the gradients may be due to reaction of the organisms to disturbance by the nets, with the added suggestion that if this is the cause, the disturbance is apparently much greater when the larger net is used.

We have also a series of ten horizontal hauls made with a single 75 cm. net, the data for which are presented in Table X. It will be observed that *Centropages typicus*, ♂, *and C. hamatus* ♀ were not enumerated in the first haul. We have allowed for this in our computations, using the iterative

procedure suggested by Snedecor (2nd Ed., 1938, § 11.5). Inspection of the data shows that we are dealing here with a peculiar situation, in that the even numbered hauls are with very few exceptions higher than the corresponding odd numbered hauls. Unfortunately there is nothing in the records to indicate the source of variation; it might, for example, have arisen if windward and leeward hauls had been alternated.

The analysis of variance table for this data follows:

	Degrees of freedom	*Sum of squares*	*Mean square*
Total	57	14.7133	
Groups	5	11.1253	
Haul:			
Between pairs	4	.7292	.1823
Odd-even	1	1.6335	1.6335
Within pairs	4	.2185	.0546
$G \times H$:			
Between pairs	20	.3105	.0155
Odd-even	5	.1419	.0284
Within pairs	18	.5544	.0308
All $G \times H$	43	1.0068	.0234

It is clear that the largest source of variability is the difference between odd and even hauls. We observe, however, that there is no indication that these differences have affected the percentage composition of the hauls, since the $G \times H$ mean squares do not show significant differences. We may therefore take the figure .0234 as an estimate of σ_{GH}^2. This figure may be compared with our previous analysis. We had then $\sigma_{GH}^2 = .0611$, $\sigma_{NGH}^2 = .0135$, whence we get .0746 as the variance comparable with our present .0234. More appropriately, perhaps, we may use the figure for σ_{GH}^2 within pairs, which was .0199, so that we have $.0199 + .0135 = 0.334$ against .0234 in the present series. We consider the agreement good.

We do not feel that an attempt to estimate σ_H^2 is warranted, in view of the peculiar characteristics of the data. It appears, however, that after allowance has been made for the difference between odd and even hauls, the order of magnitude of the haul mean square is about what would be expected.

SUMMARY AND CONCLUSIONS

1. The variation in seven series of oblique, vertical and horizontal plankton hauls, with nets of the same and of different sizes, has been studied statistically. In addition, vertical hauls from two other sources (Gardiner, 1931, and Künne, 1933) have been analyzed and compared with our own data.

2. The percentage variability was essentially the same for all the groups of animals considered. We may therefore conclude that any local irregulari-

ties in distribution (e. g., local swarming) were essentially alike for all groups.

3. One series of oblique hauls is presented, and in this series, the standard deviation of a single observation was about 31%. Eliminating haul to haul variation, there was a residual standard deviation of about 23%; the standard deviation of the haul to haul variation alone was about 20%.

4. Two series of vertical hauls of our own collection are presented. In one, the standard deviation of a single observation was about 53%. In the second, the data are insufficient to warrant a complete analysis; there is some indication of a greater haul to haul variability than in the preceding series. Gardiner's and Künne's series give standard deviations for a single observation of 43% and 47% respectively.

5. Four series of horizontal hauls are presented, two of which have been analyzed at length. The third, which is less extensive, confirms (with one exception, noted below) the indications of the other two. In these hauls, the standard deviation of a single observation is about 124%, if we wish to regard the observation as representative of conditions during a period comparable with the duration of our series (say four to five hours). A considerable part of this variation arises out of changes in the proportions of the population with time; if we eliminate these, and regard our single observation as representative of conditions during an interval comparable with that from one haul to the next (say about a half hour) the corresponding standard deviation is about 93%.

6. In the first two horizontal series, marked vertical gradients in population density occurred within the two meters sampled. In the third series, no such gradients were found. The exact nature of these gradients remains a problem for future work.

7. No significant difference in variability of catches was observed with different sizes of nets. There is indication that the smaller nets caught less than would be indicated by the ratios of their openings to that of the 75 cm. net. Thus the 30.5 cm. net caught a fraction 0.122 of the 75 cm. net catch, against an expected ratio of 0.165; and that the 12.7 cm. net caught a fraction 0.0210 against an expected 0.0285.

8. The proportions of different groups (including sagittae, amphipods and large copepods) caught by the various nets did not vary in a manner to indicate differential selection with net size, with the possible exception of *Sagitta*, as to which the evidence is not conclusive. We may therefore conclude that under the conditions of the present investigation our smallest net is as reliable for the types of plankton considered as nets of more usual size.

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APPENDIX

The Estimation of Variance Arising from Different Sources

The problem of estimating the separate components of variability occupies a central position in this paper. The method we have used is a straightforward extension of the suggestions of R. A. Fisher in his "Statistical Methods for Research Workers," § 40. We now set out the assumptions made and the procedure used.

Assume that we have data divided into k sets of m observations each. Let x_{ij} be the jth observation in the ith set. We assume that the variability of x_{ij} about some hypothetical "true value" μ is due (a) to a component of random variation common to all the members of the ith set and (b) to a component of random variation peculiar to this particular observation. We assume further that these two components of variation are independent in the probability sense, and that their effects are additive. Then we can write

$$x_{ij} = \mu + \varepsilon_i + \eta_{ij}$$

where ε and η are random variables subject to some probability law. Their mean values we assume to be zero; we wish to make estimates of their mean squares. We do not know the "true value" μ, and therefore cannot determine the deviations ε_i and η_{ij} directly. But it is shown in elementary works on probability that if $u_1, u_2, \ldots u_n$ are n values of a random variable

whose true mean is U and whose true variance is σ^2, then ($\bar{u}$ being the mean of the observed values) the mathematical expectation of

$$S\,(u_i - \bar{u})^2 = (n-1)\sigma^2.$$

Hence, if we divide (the sum of the squares of the deviations of the set from their mean) by (their total number less one) we have a quantity whose mathematical expectation is σ^2.

We may apply the same reasoning by which this result is established to the more complicated case of two categories of variation. It may readily be shown that if σ_ϵ^2 and σ_η^2 are the variances of ε and η, then the following relations hold (where $E\{\ \}$ denotes mathematical expectation):

$$E\{S_{ij}(x_{ij} - \bar{x}_i)^2\} = k(m-1)\sigma_\eta^2$$

$$E\{mS_i(\bar{x}_i - \bar{x})^2\} = (k-1)m\sigma_\epsilon^2 + (k-1)\sigma_\eta^2$$

$\bar{x}_i$ and $\bar{x}$ being the mean of the ith set and the general mean, respectively. Hence we can reach estimates of σ_η^2 and σ_ϵ^2.

For practical computation, it is convenient to note that the expressions whose expectations have been given are simply the "within sets" and "between sets" sums of squares in the ordinary analysis of variance table. The arithmetical convenience of the analysis of variance computations is great; and we have therefore set up our work in this form.

It is perhaps worth pointing out that these relations do not involve any assumption as to the form of probability distribution except those explicitly made above. In particular, no assumption of "normality" has been made. In this respect our use of the analysis of variance differs from that in common practice. The difference, of course, lies in the fact that the analysis of variance is commonly used to provide a significance test, while we use it as a means of estimation.

In more complicated situations, such as arise with more than a single classification into sets, the relations are naturally more complex; but the same basic principles govern. Suppose, for example, that we have a set of hauls, in each of which we have two nets. For concreteness, suppose 10 hauls, with 6 groups enumerated. The analysis of variance table, together with the quantities of which each mean square is an estimate, will be:

	D.F.	*Expectation of mean square*
Total	*119*	
Groups	5	
Hauls	9	$12\,\sigma_H^2 + 2\,\sigma_{GH}^2 + 6\,\sigma_{HN}^2 + \sigma_{GHN}^2$
$G \times H$	45	$2\,\sigma_{GH}^2 + \sigma_{GHN}^2$
Net	1	
$G \times N$	5	$10\,\sigma_{GN}^2 + \sigma_{GHN}^2$
$H \times N$	9	$6\,\sigma_{HN}^2 + \sigma_{GHN}^2$
$G \times H \times N$	45	σ_{GHN}^2

In building these mean squares, we have assumed that variation corresponding to each σ^2 actually exists. This assumption naturally should be tested. The use of Fisher's Z test, or Snedecor's F test, is evidently appropriate. These (equivalent) tests actually supply a criterion by which we can judge whether the existence of any particular is established by the evidence. If we decide that one or other of the σ^2 is zero or negligible,—which amounts to deciding that variation from a particular source is negligible,—we may regroup our analysis accordingly to estimate the remaining σ^2.

With data divided into sets with unequal numbers the situation is somewhat more complicated, and in extreme cases the algebra may become very heavy. For this reason, among others, it is advisable to keep the ultimate analysis in mind in planning experiments. We give here the case of N observations in k sets, the number of observations in the ith set being n_i.

The usual analysis of variance table will be:

	D.F.	*Sum of squares*
Total	$N - 1$	$S_{ij}\,(x_{ij} - \bar{x})^2$
Between sets	$k - 1$	$S_i n_i\,(\bar{x}_i - \bar{x})^2$
Within sets	$N - k$	$S_{ij}\,(x_{ij} - \bar{x}_i)^2$

It can now be shown, by straighforward but somewhat lengthy algebra, that these sums of squares have the following mathematical expectations (σ_ϵ^2 and σ_η^2 having their previous significance):

$$E\,\{S_{ij}(x_{ij} - \bar{x})^2\} = \sigma_\epsilon^2\left(N - \frac{Sn_i^2}{N}\right) + \sigma_\eta^2(N - 1)$$

$$E\,\{S_i\, n_i(\bar{x}_i - \bar{x})^2\} = \sigma_\epsilon^2\left(N - \frac{Sn_i^2}{N}\right) + \sigma_\eta^2(k - 1)$$

$$E\,\{S_{ij}(x_{ij} - \bar{x}_i)^2\} = \sigma_\eta^2(N - k)$$

From these results we can evidently get our required estimates of σ_ϵ^2 and σ_η^2.

TABLE I

SERIES I. OBLIQUE HAULS, 30 M. TO SURFACE, NET RAISED 3 M. EVERY MINUTE. 2 SCRIM (10 STRANDS/CM.) NETS, 75 CM. DIAMETER, USED IN ROTATION: AABBAA, ETC. HAULS 220A–220L. AUG. 24, 1936. LOCATION: WHISTLE BUOY AT WESTERN ENTRANCE TO VINEYARD SOUND (STATION 3). DEPTH TO BOTTOM: 38 M. FIGURES ARE ESTIMATED TOTALS IN THE CATCH. FIGURES IN PARENTHESES ARE NUMBER OF 29 CC. SAMPLES ON WHICH ESTIMATE IS BASED. DILUTION FOR C. FINMARCHICUS = 2000 CC.; FOR C. TYPICUS, 8000 CC.

Net	*C. finmarchicus* *Cop. IV*	*C. finmarchicus* *Cop. V*	*C. typicus* *adult* ♀	*C. typicus* *adult* ♂
A	(5) 895	(5) 1520	(2) 43300	(3) 11000
A	(5) 540	(5) 1610	(2) 32800	(4) 8600
B	(5) 1020	(5) 1900	(2) 28800	(4) 8260
B	(5) 470	(5) 1350	(2) 34600	(3) 9830
A	(5) 428	(5) 980	(2) 27800	(4) 7600
A	(4) 620	(4) 1710	(2) 32800	(3) 9650
B	(5) 760	(4) 1930	(2) 28100	(4) 8900
B	(5) 537	(4) 1960	(3) 18900	(5) 6060
A	(4) 845	(4) 1840	(2) 31400	(3) 10200
A	(5) 1050	(3) 2410	(2) 39500	(2) 15500
B	(5) 387	(5) 1520	(2) 29000	(4) 9250
B	(5) 497	(5) 1685	(4) 22300	(4) 7900
Mean	671	1701	30775	9396
Range	663	1480	24400	9440

TABLE II

LOGARITHMS OF ESTIMATED CATCHES. SERIES I

C.f. IV	*C.f. V*	*C.t.*, ♀	*C.t.*, ♂	*Haul Mean*
2.95	3.18	4.64	4.04	3.70
2.73	3.21	4.52	3.93	3.60
3.01	3.28	4.46	3.92	3.67
2.67	3.13	4.54	3.99	3.58
2.63	2.99	4.44	3.88	3.48
2.79	3.23	4.52	3.98	3.63
2.88	3.29	4.45	3.95	3.64
2.73	3.29	4.28	3.78	3.52
2.93	3.26	4.50	4.01	3.68
3.02	3.38	4.60	4.19	3.80
2.59	3.18	4.46	3.97	3.55
2.70	3.23	4.35	3.90	3.54
Species				
Mean 2.80	3.22	4.48	3.96	
Range .43	.39	.36	.41	

TABLE III

SERIES III. VERTICAL HAULS, 31 M. TO SURFACE AT 0.6 M./SEC. SCRIM NETS 75 CM. AND 30.5 CM. USED ALTERNATELY. HAULS 301 A–H. JUNE 29, 1937. LOCATION 4 MILES W × N OF GAY HEAD. DEPTH TO BOTTOM 31.5 M. FIGURES ARE ESTIMATED TOTALS IN THE CATCH. 1/69 OF CATCH COUNTED WITH 75 CM. NET; TOTAL CATCH COUNTED WITH 30.5 CM. NET.

	Haul	*Limacina retroversa*	*Centropagus typicus* ♀	*C. hamatus* ♀	*C. typicus* ♂	*P. minutus* ♂	*T. discaudatus* ♀
	1	1850	442	924	318	815	345
75 cm.	3	1880	940	1210	653	594	635
net	5	897	649	1710	788	525	394
	7	925	593	745	386	221	166
	2	157	86	68	43	30	35
30.5 cm.	4	247	80	192	61	66	35
net	6	184	138	196	105	40	44
	8	146	206	180	141	71	36

TABLE IV

SERIES IV. VERTICAL HAULS, 35 M. TO SURFACE AT 0.5–0.9 M./SEC. SCRIM NETS, 75 CM., 30.5 CM., AND 12.7 CM. IN DIAMETER USED IN SUCCESSION. HAULS $312A_1$–$312C_{10}$. AUG. 10, 1937. LOCATION: WHISTLE BUOY AT WESTERN ENTRANCE TO VINEYARD SOUND. DEPTH TO BOTTOM: 42 M. FIGURES ARE ESTIMATED TOTALS IN THE CATCH BASED ON INDICATED ( ) NUMBER OF 29 CC. SAMPLES. (T) INDICATES TOTAL COUNT. DILUTED FOR C. TYPICUS IN 75 CM. NET CATCH 2000 CC. IN 30.5 CM. NET CATCH 1000 CC.

	75 cm. Net (No. 2)			*30.5 cm. Net (No. 25)*			*12.7 cm. Net (No. 21)*		
	C. typicus adult ♂–♀		*S. elegans all over .5 cm. long*	*C. typicus adult* ♂–♀		*S. elegans all over .5 cm. long*	*C. typicus adult* ♂–♀		*S. elegans all over .5 cm. long*
1)	(2)	14750	(T) 123	(5)	1115	(T) 17	(T)	241	(T) 0
2)	(2)	19200	(T) 150	(4)	2010	(T) 19		341	3
3)	(2)	19640	(T) 190	(3)	2705	(T) 24		568	1
4)	(2)	8950	(T) 34	(2)	2550	(T) 7		240	0
5)	(2)	24800	(T) 63	(2)	1960	(T) 8		564	0
6)	(4)	1880	(T) 12	(5)	798	(T) 1		496	1
7)	(2)	15900	(T) 44	(2)	2150	(T) 5		334	0
8)	(4)	1850	(T) 14	(2)	2390	(T) 10		246	1
9)	(2)	13800	(T) 40	(4)	1075	(T) 3		154	0
10)	(2)	18400	(T) 90	(2)	1890	(T) 4		407	3
Total		139000	760		18600	98		3591	9

TABLE V

DATA FROM GARDINER'S TABLE II. VERTICAL HAULS, 80 M. TO SURFACE. ORGANISMS PER METER DEPTH.

Time	*Calanus*	*Temora*	*Ps. and Paracalanus*	*Other Organisms*
20:23	56	22	47	21
22:00 }]	69	13	61	31
22:05 }	46	15	55	82
22:20]	37	8	84	12
1:25 }]	36	19	47	10
1:30 }	25	15	66	16
1:37 }	98	44	159	54
1:45 }	64	38	51	66
1:55]	58	20	112	42
2:45 }]	92	15	111	43
2:54 }	139	26	132	35
3:05	139	28	142	50
3:15 }	95	31	193	61
3:23 }]	155	50	134	40
3:50 }]	95	36	169	52
3:55 }	159	61	139	85
4:02 }	156	43	206	87
4:08 }	150	49	208	89
4:15 }	166	57	237	84
4:20 }	91	46	229	40
4:33]	101	43	161	62
5:00 }]	89	46	73	38
5:07 }	111	59	77	28
5:15 }	72	61	102	44
5:20 }	90	30	89	56
5:25 }	108	45	154	49
5:35 }]	151	53	132	48
6:45	104	44	192	54
7:00 }]	101	71	154	26
7:08 }	146	84	231	78
7:12 }	113	74	199	72
7:20 }	115	62	137	51
7:30 }	111	58	234	58
7:36 }	143	44	169	48
7:45 }	114	50	151	66
7:50 }]	102	47	164	78

} Grouping in 10 minute intervals.

] Grouping in hour intervals.

TABLE VI

(From Künne, 1933, Table 8.) Vertical Hauls; H = Hensen Egg Net; L = Helgoland Larva Net. Figures Are Total Counts.

Series	Net	*Small Medusa*	*Terebellid larvae*	*Tomopteris*	*Calanus*	*Decapod larvae*
62	H	354	8	54	97	51
	L	1521	22	200	1275	1398
	L	1451	19	257	618	603
	H	489	10	63	182	146
	H	375	12	26	143	107
63	L	1384	23	193	745	411
	L	1414	36	113	742	566
	H	473	3	31	195	223
	H	437	2	30	182	144
64	L	1769	26	104	852	857
	L	1948	31	92	958	820
	H	442	4	29	210	264
	H	326	4	24	155	94
65	L	1437	24	92	566	325
	L	1522	20	63	551	318
	H	415	7	39	163	141

		Small Medusae	*Calanus*	*Decapod larvae*	*Fish Eggs*
	H	29	26	24	64
73	L	127	79	43	298
	L	92	90	73	301
	H	21	32	26	68
	H	19	22	21	90
74	L	85	119	132	340
	L	92	114	144	224
	H	27	34	21	72
	H	29	30	46	80
75	L	35	93	160	252
	L	45	115	178	360
	H	11	23	33	74
	H	6	20	48	53
76	L	56	120	231	251
	L	63	106	247	221
	H	11	34	55	51

TABLE VII

SERIES V. HORIZONTAL HAULS, 15 MIN. AT 2 KNOTS. SCRIM NETS 12.7 CM. IN DIAMETER (NOS. 22 AND 23) AT 29 AND 31 M., AND SCRIM NET 75 CM. IN DIAMETER (NO. 2) AT 30 M. TOWED SIMULTANEOUSLY. HAULS $324A_1$–$324C_{10}$. AUGUST 26, 1937. LOCATION: WHISTLE BUOY NO. 6 OFF CHATHAM. DEPTH TO BOTTOM: 35 N. FIGURES ARE ESTIMATED TOTALS IN THE CATCH BASED ON INDICATED ( ) NUMBER OF SAMPLES. (T) = TOTAL COUNT. FOR NETS NOS. 22 AND 23: DILUTION 1000 CC. SAMPLE SIZE 58 CC.; FOR NET NO. 2 DILUTION 10,000 CC. SAMPLE SIZE 29 CC.

	Calanus finmarchicus cop. V	*C. finmarchicus adult* ♀	*Metridia lucens adult* ♀	*Centropages typicus adult* ♂ ♀	*Euthemisto juv.*
No. 22	860 (4)	400 (4)	492 (4)	1100 (4)	48 (4)
No. 2	50500 (2)	21000 (2)	17400 (2)	55500 (2)	2490 (5)
No. 23	403 (5)	217 (5)	259 (5)	1150 (2)	15 (T)
No. 23	1110 (2)	266 (5)	855 (3)	810 (3)	87 (5)
No. 2	54500 (2)	15800 (2)	33000 (2)	25200 (2)	2800 (5)
No. 22	450 (4)	135 (5)	380 (5)	450 (4)	24 (5)
No. 22	1400 (2)	218 (5)	1850 (2)	790 (3)	148 (5)
No. 2	71000 (2)	8950 (5)	79000 (2)	25500 (2)	2700 (5)
No. 23	1050 (2)	156 (5)	1670 (2)	760 (3)	103 (5)
No. 23	344 (5)	45 (5)	387 (5)	278 (5)	93 (5)
No. 2	27800 (2)	4950 (5)	32000 (2)	19600 (2)	2420 (5)
No. 22	366 (5)	66 (5)	545 (4)	485 (4)	58 (5)
No. 22	442 (5)	79 (5)	183 (5)	705 (3)	48 (5)
No. 2	19800 (2)	3030 (5)	13200 (3)	33500 (2)	965 (5)
No. 23	200 (5)	41 (5)	141 (5)	560 (4)	14 (5)
No. 23	705 (3)	120 (5)	960 (3)	240 (5)	65 (5)
No. 2	81000 (2)	10500 (4)	72000 (2)	20500 (2)	4400 (5)
No. 22	1440 (2)	131 (5)	1310 (2)	495 (4)	93 (5)
No. 22	1170 (2)	110 (5)	5780 (2)	920 (3)	65 (5)
No. 2	59700 (2)	6230 (5)	203000 (2)	20700 (2)	1070 (5)
No. 23	1200 (2)	107 (5)	4350 (2)	297 (5)	59 (5)
No. 23	276 (5)	38 (5)	1060 (2)	480 (4)	55 (5)
No. 2	18300 (2)	1660 (5)	59000 (2)	20300 (2)	670 (5)
No. 22	380 (5)	45 (5)	1090 (2)	410 (5)	35 (5)
No. 22	965 (2)	72 (5)	1900 (2)	396 (5)	90 (5)
No. 2	46000 (2)	4200 (5)	57500 (2)	13600 (3)	1930 (5)
No. 23	835 (3)	69 (5)	805 (3)	350 (5)	38 (5)
No. 23	1060 (2)	114 (5)	400 (5)	440 (4)	55 (5)
No. 2	52500 (2)	5050 (5)	17100 (3)	17600 (2)	2500 (5)
No. 22	610 (3)	41 (5)	332 (4)	245 (5)	28 (5)

TABLE VIII

SERIES VI. HORIZONTAL HAULS, 10 MIN. AT 2 KNOTS. SCRIM NETS 12.7 CM. IN DIAMETER (NO. 21 WITH VOLUME METER AND NO. 23) AT 29 AND 31 M., AND SCRIM NET 75 CM. IN DIAMETER (NO. 2) AT 30 M. TOWED SIMULTANEOUSLY. HAULS $327A_1$–$327C_{11}$. SEPTEMBER 9, 1937. LOCATION: CAPE COD BAY, OFF CANAL ENTRANCE. DEPTH TO BOTTOM 36 M. FIGURES ARE ESTIMATED TOTALS IN THE CATCH BASED ON INDICATED ( ) NUMBER OF SAMPLES. DILUTIONS AND SAMPLES AS IN SERIES V.

	C. finmarchicus *cop. V.*	*C. finmarchicus* *adult ♀*	*Metridia lucens*	*C. typicus* ♂ ♀	*Euthemisto juv.*	*Sagitta elegans*	*Volume meter*
No. 21	1730 (4)	169 (5)	36 (8)	194 (5)	235 (5)	45 (T)	—
No. 2	81500 (4)	8300 (4)	2760 (5)	9500 (4)	19900 (4)	3020 (4)	
No. 23	684 (7)	69 (5)	65 (5)	72 (5)	118 (5)	7 (5)	
No. 23	3200 (2)	303 (5)	59 (5)	149 (5)	905 (2)	100 (2)	
No. 2	84500 (2)	7000 (5)	2760 (5)	9150 (4)	12800 (4)	4350 (5)	
No. 21	1340 (2)	110 (5)	260 (5)	230 (5)	155 (5)	62 (5)	1384
No. 21	1160 (2)	76 (5)	41 (5)	256 (5)	124 (5)	117 (5)	—
No. 2	51500 (2)	2700 (5)	3700 (5)	6500 (5)	9650 (4)	6060 (5)	
No. 23	1220 (2)	73 (4)	263 (4)	164 (4)	152 (5)	76 (5)	
No. 23	990 (2)	58 (5)	79 (5)	169 (5)	259 (5)	110 (5)	
No. 2	42300 (2)	2480 (5)	18300 (2)	6200 (5)	4420 (5)	6600 (5)	
No. 21	785 (3)	35 (5)	605 (3)	276 (5)	107 (5)	65 (5)	1305
No. 21	1725 (2)	176 (5)	62 (3)	228 (5)	262 (5)	100 (5)	1644
No. 2	Haul lost						
No. 23	1940 (2)	166 (5)	89 (5)	114 (5)	272 (3)	48 (5)	
No. 21	735 (3)	79 (5)	14 (5)	169 (5)	180 (5)	27 (5)	1334
No. 2	36600 (3)	2280 (5)	345 (5)	5500 (5)	5100 (5)	1240 (5)	
No. 23	930 (2)	79 (5)	45 (5)	117 (5)	280 (5)	27 (5)	
No. 23	1200 (2)	145 (5)	17 (5)	104 (3)	230 (5)	31 (5)	
No. 2	29400 (2)	1860 (5)	895 (5)	4200 (5)	5050 (5)	1240 (5)	
No. 21	706 (3)	38 (5)	45 (5)	121 (5)	193 (5)	52 (5)	1392
No. 21	1350 (2)	73 (5)	204 (5)	280 (5)	320 (5)	20 (5)	1531
No. 2	49000 (2)	3100 (5)	12400 (3)	5450 (5)	11700 (4)	1040 (5)	
No. 23	690 (3)	41 (5)	390 (5)	117 (5)	250 (5)	7 (T)	
No. 23	950 (2)	97 (5)	190 (5)	114 (5)	328 (5)	24 (5)	
No. 2	38500 (2)	2140 (5)	12600 (3)	4700 (5)	10600 (4)	415 (5)	
No. 21	575 (3)	38 (5)	760 (3)	169 (5)	120 (5)	6 (T)	1372
No. 21	1430 (2)	90 (5)	262 (5)	131 (5)	500 (5)	27 (T)	1741
No. 2	59000 (2)	1725 (5)	22800 (5)	3700 (3)	19000 (3)	690 (4)	
No. 23	1070 (2)	79 (5)	880 (2)	107 (5)	354 (5)	5 (T)	
No. 23	Haul lost						
No. 2	49000 (2)	1725 (5)	1930 (5)	4550 (5)	7050 (5)	3040 (5)	
No. 21	1060 (2)	62 (5)	338 (5)	159 (5)	148 (5)	41 (5)	1131

TABLE IX

SERIES VII. HORIZONTAL HAULS. SCRIM NETS 12.7 CM. IN DIAMETER (NO. 21 WITH VOLUME METER AND NO. 23) TOWED SIMULTANEOUSLY AT 29 AND 30 M. SERIES MADE IMMEDIATELY AFTER SERIES VI, SAME PROCEDURE

Net	*Calanus* cop. *V*	*Calanus* adult ♀	*Metridia* ♀	*C. typicus* ♂ ♀	*Euthemista*	*Sagitta*
21	1320 (2)	93 (5)	21 (5)	190 (5)	117 (5)	69 (5)
23	1130 (2)	152 (5)	14 (5)	121 (5)	190 (5)	59 (5)
23	563 (3)	45 (2)	14 (5)	179 (5)	79 (5)	20 (T)
21	934 (2)	72 (5)	31 (5)	193 (5)	114 (5)	38 (5)
21	493 (4)	14 (5)	124 (5)	293 (5)	110 (5)	7 (T)
23	290 (5)	24 (5)	55 (5)	134 (5)	48 (5)	3 (T)
23	193 (5)	17 (5)	221 (5)	162 (5)	131 (5)	1 (T)
21	586 (3)	34 (5)	355 (5)	217 (5)	93 (5)	5 (T)
21	586 (3)	45 (5)	217 (5)	121 (5)	97 (5)	5 (T)
23	598 (3)	24 (5)	286 (5)	176 (5)	103 (5)	8 (T)

TABLE X

SERIES II. HORIZONTAL HAULS AT 13 M. ONE OPEN SCRIM NET (10 STRANDS/CM.), 75 CM. DIAMETER. HAULS 133 A–J. AUGUST 9, 1935. LOCATION: WHISTLE BUOY AT WESTERN ENTRANCE TO VINEYARD SOUND (STATION 3). DEPTH TO BOTTOM: 38 M. FIGURES ARE ESTIMATED TOTALS IN THE CATCH. FIGURES IN PARENTHESES ARE NUMBER OF 29 CC. SAMPLES ON WHICH ESTIMATE IS BASED. DILUTION FOR C. FINMARCHICUS = 3000 CC., EXCEPT IN ONE INSTANCE (133B. C. FINMARCHICUS V, DIL. = 6000 CC.) FOR C. TYPICUS AND C. HAMATUS = 300 CC.

Calanus finmarchicus			*Centropages typicus*		*Centropages hamatus*
IV	*V*	adult ♀	adult ♀	adult ♂	adult ♀
(4) 3410	(2) 12900	(4) 1110	(4) 1880	(a)	(a)
(2) 8700	(2) 14700	(5) 825	(5) 2410	(5) 845	(5) 845
(4) 2780	(3) 4180	(5) 415	(5) 1300	(5) 165	(5) 226
(3) 4750	(2) 8550	(5) 950	(5) 2000	(5) 868	(5) 279
(5) 1300	(5) 2080	(5) 290	(5) 847	(5) 394	(5) 186
(3) 4200	(2) 9250	(5) 1135	(5) 1780	(5) 620	(5) 600
(5) 2140	(5) 3200	(5) 370	(5) 1160	(5) 310	(5) 206
(3) 4800	(2) 5840	(5) 535	(5) 1740	(5) 890	(5) 1390
(5) 1590	(5) 4800	(5) 725	(5) 1034	(5) 165	(5) 330
(2) 5740	(2) 6350	(5) 660	(4) 2800	(5) 1300	(5) 745

(a) Not enumerated.

THE MOTION OF ATMOSPHERIC DISTURBANCES

By

B. HAURWITZ†

In a recent paper Rossby has discussed the effect of the latitudinal variation of the Coriolis force on the propagation of oceanic and atmospheric disturbances (*2*). He showed that the velocity c of a disturbance in an incompressible atmosphere on a plane earth when only the effect of the latitudinal variation of the Coriolis force is considered is given with sufficient accuracy by the formula

$$c = U - \frac{\beta L^2}{4\pi^2} \tag{1}$$

Here U denotes the undisturbed velocity of the air, that is the geostrophic wind, L the wave length of the disturbance. β is the meridional variation of the Coriolis parameter,

$$\beta = \frac{\partial f}{\partial y} = \frac{2\omega \cos \varphi}{a} \tag{2}$$

Here ω is the angular velocity of the earth's rotation, a the earth's radius, φ the geographic latitude. The direction of the wave propagation is assumed to coincide with the W–E direction which serves as x-axis. Rossby assumes further that the disturbance is independent of the y-coordinate, that is of the direction normal to the direction of propagation. His investigation has been extended in three particulars. These extensions may prove helpful in future practical work.

1) It will not be assumed that the disturbance is independent of the y-coordinate, for that assumption implies infinite lateral extent of the disturbance. Such is not the case in the earth's atmosphere. Since the dimensions of the disturbance may be rather large fractions of the earth's circumference as will be seen later (p. 41) it would appear desirable to take into account the spherical shape of the earth. This extension will be given in a later paper. The mathematical treatment of the spherical case leads to rather complicated formulae. It seems therefore worth while to extend the discussion as far as possible by elementary mathematical methods.

2) The effect of friction will be considered. It will be assumed that the effect of friction on the vorticity is proportional to the vorticity itself.

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This assumption is admittedly rather crude, but it will at least show qualitatively how the motion is influenced by friction.

3) Finally the action of a force on the vorticity will be considered. This force may, for instance, be due to the concentration of solenoids along the coast lines as pointed out by Rossby (*2*). He has already indicated how the action of such a force may be taken into account by imposing a boundary condition on the solution so that velocity and vorticity have prescribed values in the region where the force is located. To obtain a clearer picture of the dynamical process, however, it is necessary to have the expression for the force appear in the dynamic equations. The force starts to act only at the moment when the air passes over the region where it is concentrated. Therefore the forced oscillation will not be completely developed at the moment when the region of the force is passed but somewhat later.

Reference is here made only to atmospheric disturbances but the application to oceanic motions is obvious.

A) Disturbance of finite lateral extent

A simple expression for the pressure distribution of a disturbance of finite lateral extent is given by

$$p = \text{const.} - A \cdot y + B \cos \frac{2\pi}{D} y \cos \frac{2\pi}{L} x \tag{3}$$

where D represents the width in the y-direction of a High and a successive Low. The pressure field defined by (3) gives a succession of Highs and Lows in the y-direction as well as in the x-direction. The smaller the undisturbed meridional pressure gradient A is compared to the intensity of the superimposed disturbance B the greater is the number of closed isobars. When the ratio A/B increases fewer isobars are closed and the pressure field resembles more closely a succession of wedges and troughs. A small value of the ratio A/B corresponds to the actual atmospheric conditions near the earth's surface where closed Highs and Lows are observed. The intensity B of these disturbances decreases upwards while the undisturbed meridional pressure gradient A remains of the same magnitude. Consequently most of the isobars at upper levels resemble cosine curves with few if any isobars closed.

If only one High or Low is to be considered, the right hand side of (3) may be represented by a different function which can be expressed by a Fourier double integral.

When the waves are of finite lateral extent the velocity of propagation is found to be considerably different from that obtained when the waves are of infinite lateral extent. The motion considered may be purely horizontal,

without friction, and the atmosphere may be incompressible and homogeneous as in the case treated by Rossby. It follows from the equations of motion that the absolute vorticity which consists of the sum of the vorticity ζ of the motion relative to the earth and of the vorticity f due to the rotation of the earth must be constant,

$$\zeta + f = \text{const.} \tag{4}$$

Obviously, only the vertical component of the vorticity

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

has to be considered in this case of horizontal motion. The equation of continuity for horizontal incompressible motion states that

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{5}$$

Differentiation of (4) shows, since f depends only on the latitude, that

$$\frac{d\zeta}{dt} = -\beta v \tag{6}$$

where β is defined by (2). Owing to the meridional pressure gradient a geostrophic zonal current of the velocity U must exist. On this zonal current a disturbance with the velocity components u' and v' is superimposed. u' and v' may be considered so small that terms of higher order in the perturbation velocities or their derivatives can be neglected. It follows then from (6) and (5) that

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\left(\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}\right) = -\beta v' \tag{7}$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0 \tag{8}$$

The quantity β depends on the latitude. However, as long as the earth is regarded as a plane this variability of β may be neglected since the error involved although considerable is of the same order as the one caused by considering the earth as flat.

The form of eq. (8) suggests that the streamfunction ψ may advantageously be introduced

$$u' = -\frac{\partial \psi}{\partial y} \qquad v' = \frac{\partial \psi}{\partial x} \tag{9}$$

The equation $\psi = \text{const.}$ represents the stream lines.

The equation of continuity is satisfied for any value of ψ. The vertical component of the curl is given by

$$\zeta = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} \tag{10}$$

Using eq. (9) eq. (7) is transformed into

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) = -\beta\frac{\partial\psi}{\partial x}. \tag{11}$$

A solution may be assumed in the form

$$\psi = C\cos\frac{2\pi}{D}y\sin\frac{2\pi}{L}(x - ct) \tag{12}$$

where C is an arbitrary constant. D represents the width of the disturbance, L its wave length and c the velocity of the disturbance. Substituting (12) in (11) it follows that

$$c = U - \frac{\beta L^2}{4\pi^2}\frac{D^2}{D^2 + L^2}. \tag{13}$$

If the motion is not a function of the y-coordinate, $D = \infty$, and (13) becomes identical with (1). If D is finite, $\dfrac{1}{1 + L^2/D^2}$ is smaller than unity. Thus the smaller the lateral extent of the disturbance, the smaller is the influence of the term which is due to the latitudinal variation of the Coriolis force. The value of this factor for different ratios L/D is shown in Table I. When the lateral extent of the disturbance is equal to the wave length, for

TABLE I

Reduction of the Velocity Deficit U-c when It Is Assumed that the Disturbance Is of Finite Lateral Extent

L/D	1/5	1/3	1	2	3
$\dfrac{1}{1+L^2/D^2}$	.96	.90	.50	.20	.10

instance, the difference between the geostrophic velocity and the velocity of the perturbation is reduced to half the value which is found for an infinite disturbance.

The disturbance remains stationary, $c = 0$, when

$$U = \frac{\beta}{4\pi^2}L^2\frac{1}{1 + L^2/D^2} \tag{14}$$

The length of the stationary wave L_s, for a certain wind velocity U is therefore given by

$$L_s = 2\pi\sqrt{\frac{U}{\beta(1 - \frac{4\pi^2}{\beta} U/D^2)}} \tag{15}$$

or

$$L_s = 2\pi\sqrt{\frac{U + (1 + L_s^2/D^2)}{\beta}} \tag{16}$$

if the ratio L_s/D is given, The length of the stationary wave increases as L_s/D increases. If, for instance, $L_s/D = 1$, L_s increases 41% above the value when $D = \infty$. If $L_s/D = \frac{1}{2}$ the increase is only 11%. The effect of the finite lateral extent of the disturbances is therefore to diminish the number of possible stationary disturbances around the circumference of the earth at a given latitude while increasing the size of the perturbation.

Equation (15) shows that for perturbations of finite lateral extent a stationary wave length exists only if

$$D > 2\pi\sqrt{U/\beta}$$

that is when the lateral extent of the perturbation is greater than the stationary wave length for a perturbation whose lateral extent is infinite.

When $D = \infty$ (15) and (16) reduce to the expression

$$L_s = 2\pi\sqrt{U/\beta}$$

which was given by Rossby. When the value of β from (2) and the angular velocity of the air motion relative to the earth, given by

$$\dot{\lambda} = \frac{U}{a \cos \varphi},$$

are introduced here it follows that

$$L_s = \pi a \sqrt{\frac{2\dot{\lambda}}{\omega}} \tag{17}$$

The only variable in this equation is $\dot{\lambda}$. The table published by Rossby (*2*) shows that the maximum angular velocity during February is approximately constant at least from 70° to 30° northern latitude, its value being 5.02×10^{-6} sec^{-1}. This would give a stationary wave length of 7550 km. To show this graphically lines of equal wind velocity have been plotted in Fig. 1 with the geographical latitude as abscissa and the stationary wave length of the disturbances of finite lateral extent as ordinate. The circles

indicate the position of the wind velocities given in Rossby's table. They lie fairly closely on a straight line corresponding to a stationary wave length of about 7500 km. According to (16) these considerations concerning

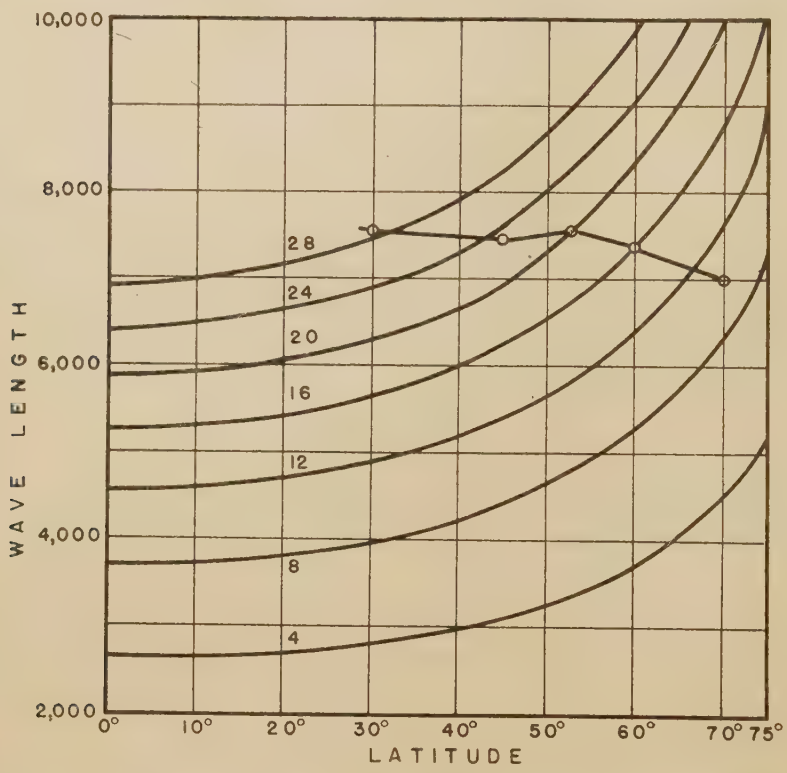

Figure 1. Stationary wave lengths of disturbances of infinite lateral extent. The curves are lines of equal wind velocity in m/sec. The circles indicate the zonal wind velocities at different latitudes on the northern hemisphere.

the constancy of L_s when $\dot{\lambda}$ is constant also hold when the disturbances are of finite lateral extent, provided that the ratio L_s/D is the same everywhere.

Table II and Fig. 2 show the stationary wave lengths for different wind velocities and latitudes when the perturbation extends from the pole to the equator. This value assumed for the magnitude of the lateral extent of the perturbation may be rather an extreme one but for a smaller lateral extent the effect is still greater. For long waves the deviations from the infinite case treated by Rossby become quite large. At latitude 60°, for instance,

the stationary wave length is almost 15000 km. as compared to 8300 km. in the infinite case.*

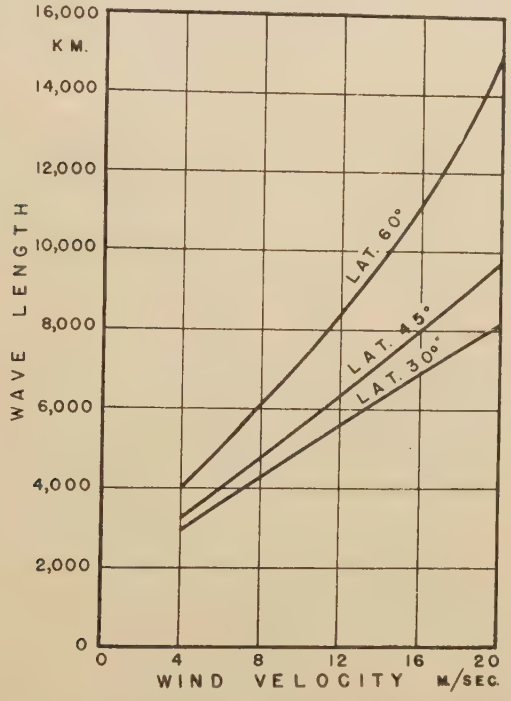

Figure 2. Stationary wave lengths when the perturbation extends from the pole to the equator.

TABLE II

STATIONARY WAVE LENGTH WHEN THE PERTURBATION EXTENDS FROM POLE TO EQUATOR

φ \ U	*4 m/sec*	*8 m/sec*	*12 m/sec*	*16 m/sec*	*20 m/sec*
30°	2940 km.	4350 km.	5600 km.	6850 km.	8120 km.
45°	3280	4820	6440	8000	9750
60°	4000	6160	8410	11100	14980

* It should however, be understood that the approximation involved in the derivation of these equations is not very good in the case of such long waves. To get more accurate information it will be necessary to take into account the spherical form of the earth.

A more general solution of (11) than the one represented by (13) can be obtained by Cauchy's method. It can be made to satisfy the initial condition that at the time $t = 0$ the stream function is equal to a given function

$$\psi_{t=0} = F(x, y)$$

The solution is then

$$(18)\quad \psi = \frac{1}{\pi^2}\int_0^{\infty}\int \int_{-\infty}^{+\infty}\int F(\sigma,\tau)\cos\lambda\left[x-\sigma-\left(U-\frac{\beta}{\lambda^2+\mu^2}\right)t\right] \cos\mu(y-\tau)\,d\sigma d\tau d\lambda d\mu$$

Here λ, μ, σ and τ are variables of integration. Equation (18) shows that each harmonic component moves with a different velocity

$$U - \frac{\beta}{\lambda^2 + \mu^2}$$

depending on the wave length $2\pi/\lambda$ and the lateral extent $2\pi/\mu$ of the harmonic. The observations show the integrated effect of all the harmonics which is represented by (18). Sufficiently large harmonics move towards the west, sufficiently small ones towards the east, so that the disturbance may split into two parts moving in opposite directions as Rossby has already pointed out. It is, of course, not necessary that the motion of the harmonics should take place in opposite directions. Even if the different harmonics proceed in the same direction with different velocities the disturbance may break up. Whether or not this split will occur depends on the initial distribution $F(x, y)$ of the disturbance. If this is a function such that only the harmonic components in the vicinity of a certain velocity have appreciable amplitudes while the amplitudes of the others are small, no separation will be observed on the map. In this case the disturbance will spread over a larger area and its intensity will weaken.

B. The equations when friction and external forces are acting

The perturbations which have been discussed so far are free perturbations which weaken gradually owing to the influence of friction. The effect of friction will be assumed directly proportional to the intensity of the vorticity. Thus

$$\text{frictional force} = -k\zeta$$

where k is the coefficient of friction. New perturbations may be generated by external forces, for instance by the action of solenoids distributed along the boundary of land and ocean. For the sake of simplicity it may be assumed that the force depends on the x-coordinate only, that is on the longitude, and is independent of the latitude. These assumptions about the

friction and the external force will at least give an approximate idea as to the effect of these two factors.

Furthermore it will be assumed now that an external force $K(x)$ is acting which generates solenoids and which depends only on the x-coordinate. The equation for the vorticity becomes

$$\frac{d\zeta}{dt} + k\zeta + \beta v = K(x). \tag{19}$$

As previously the total velocity may be the sum of the undisturbed (geostrophic) velocity U along the x-axis (directed from W to E) and of the perturbation velocity with the components u' and v'. Owing to the smallness of u' and v' (19) may be simplified so that

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x} + k\right)\left(\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}\right) + \beta v' = K(x). \tag{20}$$

Since the equation of continuity (8) remains unchanged a stream function ψ can be introduced again by (9). Equation (20) is then transformed into

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x} + k\right)\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) + \beta\frac{\partial\psi}{\partial x} = K(x). \tag{21}$$

The solution will be assumed in the form

$$\psi = Ae^{i(\mu x - \nu t)}\, e^{i\lambda y} + g(x). \tag{22}$$

A is an arbitrary constant, representing the amplitude of the disturbance, the wave length $L = 2\pi/\mu$, and the width of the disturbance $D = 2\pi/\lambda$. The time factor ν has to be determined by substituting (22) in (21). The calculations are simpler when the solution is assumed in complex rather than in real form. To obtain a physical interpretation either the real or the imaginary part of the complex solution or a linear combination of both may be chosen since each part separately represents a solution of the differential equation (21). The function $g(x)$ depends on the x-coordinate only. It represents the influence of the external force $K(x)$ on the perturbation. Substituting (22) in (21) the following two relations are obtained

$$\nu = \mu\left(U - \frac{\beta}{\mu^2 + \lambda^2}\right) - ik \tag{23}$$

and

$$\frac{d^3g}{dx^3} + \frac{k}{U}\frac{d^2g}{dx^2} + \frac{\beta}{U}\frac{dg}{dx} = \frac{K(x)}{U}. \tag{24}$$

Equation (23) gives the relation between the time factor ν and the dimensions of the disturbance which are denoted by μ and λ. When the friction

is not taken into consideration $k = 0$ and (23) is identical with (13) since the wave velocity $c = \nu/\mu$. Substituting (23) in (22) and retaining only the real part of the solution* ψ may be written

$$(25)\quad \psi = A \cos\frac{2\pi}{L}\left[x - \left(U - \frac{\beta L^2}{4\pi^2}\,\frac{D^2}{D^2 + L^2}\right)t\right] e^{-kt} \cos\frac{2\pi}{D}\,y + g(x)$$

C. Friction

When $K(x) = 0$ the function $g(x)$ may be omitted and the first term of (25) represents the complete solution. This expression shows that the velocity of propagation of the free perturbation remains the same as in the simpler case (eq. 13) when the effect of friction was not considered. Friction causes the amplitude of the disturbance to decrease exponentially with time.

The order of magnitude of the coefficient of friction k has been determined as $10^{-5} - 10^{-4}$ sec^{-1} from surface observations (*1*). This value is probably much too high if the whole atmosphere is considered instead of the surface layers only. Here the values 10^{-6} sec^{-1} and $5 \cdot 10^{-6}$ sec^{-1} have been chosen for k in order to obtain a first approximation to the rate of dissipation of the disturbances. The corresponding values of e^{-kt}, i. e. the fraction by which the amplitude is diminished after a certain number of days, are given in Table III. With the smaller value of k the amplitude would be reduced to 50% of its original value after 8 days, with the larger one in less than two days.

TABLE III

RATE OF DISSIPATION OF ATMOSPHERIC DISTURBANCES WITH TIME; $k = 10^{-6}$ SEC^{-1} AND $k = 5 \cdot 10^{-6}$ SEC^{-1}

Time in days	*2*	*4*	*6*	*8*	*10*
$k = 10^{-6}$ sec^{-1}	.84	.71	.60	.50	.42
$k = 5 \times 10^{-6}$ sec^{-1}	.42	.18	.075	.031	.013

For stationary waves the time factor ν in (22) is zero. Neglecting for the present the influence of the external force $K(x)$ the solution of (21) is given by

$$(26)\qquad \psi = e^{-\frac{k}{2U}x}\,(A_1 \cos \sigma x + A_2 \sin \sigma x)$$

or

* More accurately $A = a + ib$ should be regarded as a complex constant giving a solution of the form $(a + ib)e^{i(\mu x - \nu t)} = (a + ib)\,[\cos(\mu x - \nu t) + i \sin(\mu x - \nu t)]$. The real part of this expression is $a \cos(\mu x - \nu t) - b \sin(\mu x - \nu t) = A' \cos(\mu x - \nu t - \alpha)$ where A' and α are given by a and b. The difference between this expression and the one given in the text is, however, obviously not significant.

(27) $$\psi = Ae^{-\frac{k}{2U}x}\cos(\sigma x - \gamma).$$

Here A_1 and A_2 or A and γ are arbitrary constants of integration. The wave length $L_s = 2\pi/\sigma$ is given by the following expression

$$\sigma = \sqrt{\frac{\beta}{U} - \frac{k^2}{4U^2}}.$$

Therefore

(28) $$L_s = \frac{2\pi}{\sqrt{\beta/U - k^2/4U^2}}.$$

Equation (28) shows that the wave length of the stationary wave increases with increasing friction. For the zonal wind velocity the inequality

(29) $$U > \frac{k^2}{4\beta}$$

must hold. Otherwise σ becomes imaginary and ψ loses its wave character. Even with the high value $k = 5 \cdot 10^{-6}$ and with $\beta = 1.619 \cdot 10^{-11}$ m^{-1} sec^{-1} (at 45° lat.), for instance, the lower limit for U becomes very small, about 0.4 m/sec so that the presence of friction in general will not affect the possibility of the formation of stationary waves. Similarly equation (28) shows that the effect of friction on the wave length is small.

The values of the stationary wave length for different latitudes and wind velocities are given in Table IV where it is assumed that $k = 10^{-6}$. These figures should be compared with those given by Rossby (*2*). It will be seen that the difference in the wave lengths is negligible.

TABLE IV

Stationary Wave Lengths in km. When the Coefficient of Friction $k = 10^{-6}$

φ \ U	*4*	*8*	*12*	*16*	*20 m/sec.*
30°	2830 km.	4000 km.	4890 km.	5650 km.	6320 km.
45°	3130	4420	5410	6250	6990
60°	3720	5260	6440	7430	8320

The main effect of friction is to produce a decrease in the amplitude of the stationary disturbance with increasing distance from the origin. Table V shows the decrease of the amplitude for the same latitudes and wind velocities as in Table IV and at a distance from the origin which is equal to twice the stationary wave length. The decrease in the amplitude per wave length is greater for smaller than for larger wind velocities even though for

TABLE V

THE DECREASE OF THE AMPLITUDE OF THE PERTURBATION OVER A DISTANCE EQUAL TO TWICE THE STATIONARY WAVE LENGTH. ORIGINAL AMPLITUDE = 1. $k = 10^{-6}$

φ \ U	4	8	12	16	20 m/sec.
30°	.494	.607	.665	.695	.729
45°	.457	.578	.636	.669	.705
60°	.394	.519	.584	.620	.660

the latter the wave lengths increase. Consequently it may be expected that the larger stationary perturbation patterns repeat themselves while the smaller ones will have a tendency to appear but once or with only a few repetitions. They will not tend to spread around the whole circumference.

According to (10) and (27) the vorticity is given by

$$\zeta = \frac{\partial^2 \psi}{\partial x^2} = -\frac{A}{U}\sqrt{k^2\sigma^2 + \beta^2}\, e^{-\frac{k}{2U}x} \cos(\sigma x - \gamma + \delta) \tag{30}$$

where

$$\tan \delta = \frac{k\sigma}{\beta}.$$

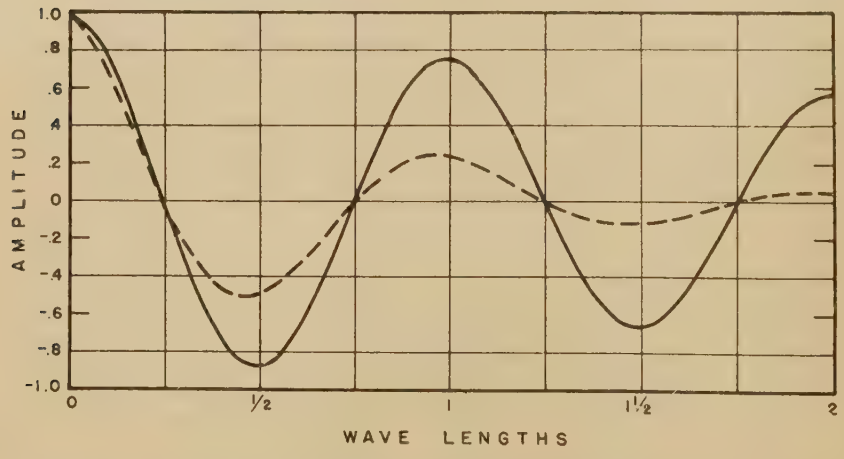

Figure 3. The vorticity distribution in a damped stationary perturbation. Lat. 45°, $U = 8$ m/sec. $k = 10^{-6}$ sec^{-1} (full curve) and $5 \cdot 10^{-6}$ sec^{-1} (broken curve).

The distribution of ζ along the x-axis in a damped stationary disturbance is represented in Fig. 3. The integration constants A and γ have been chosen so that

$$-\frac{A}{U}\sqrt{k^2\sigma^2+\beta^2}=1$$

and

$$\gamma=\delta$$

It has furthermore been assumed that $\varphi = 45°$, $U = 8$ m/sec, $k = 10^{-6}$ sec^{-1} in one case and $k = 5 \cdot 10^{-6}$ sec^{-1} in the other. When $k = 10^{-6}$ sec^{-1} the length of the stationary wave is 4420 km. When $k = 5 \cdot 10^{-6}$ sec^{-1} it is 4530 km. so that the effect of k on the wave length is small. The effect of friction on the amplitude is small when $k = 10^{-6}$ sec^{-1}. But when $k = 5 \cdot 10^{-6}$ sec^{-1} the damping already becomes very noticeable at a distance of half a wave length from the origin (broken curve).

D. External Force

The action of the external force $K(x)$ is represented by the function $g(x)$ in (25) which has to be determined from (24). This equation is similar to the equation for forced oscillations subjected to damping. The main difference is that (24) is a third order equation. But since

$$v'=\frac{\partial\psi}{\partial x}=\frac{dg}{dx} \tag{31}$$

the equation (24) may be written as a second order differential equation in v'. The appearance of the third derivative of g is explained physically by the fact that the external force $K(x)$ acts on the changes of the vorticity, not on the velocity directly. Another difference is the appearance of the length coordinate as the independent variable instead of the time. This occurs because the external force depends on the x-coordinate. The case of an external force varying with time could, of course, be treated in a similar manner.

The solution of (24) is given by

$$v'=\frac{e^{-\frac{k}{2U}x}}{\sigma U}\left[I_1(x)\sin\sigma x-I_2(x)\cos\sigma x\right] \tag{32}$$

where

$$I_1(x)=\int_0^x K(x)\,e^{-\frac{k}{2U}x}\cos\sigma x\,dx \tag{33a}$$

$$I_2(x)=\int_0^x K(x)\,e^{-\frac{k}{2U}x}\sin\sigma x\,dx \tag{33b}$$

The lower limits of the integrals $I_1(x)$ and $I_2(x)$ have been chosen so that $v' = 0$ when $x = 0$. The forced perturbation starts at the longitude defined by $x = 0$.

The solenoids which generate vorticity are frequently concentrated in a narrow strip near the coast and do not extend very far across the continent and the ocean. It may therefore be assumed that $K(x) \neq 0$ only in the interval $0 < x < \varepsilon$ and that $K(x) = 0$ everywhere else. It follows then from (33a) and (33b) that

$$\left.\begin{aligned} I_1(x) &= I_1(\varepsilon) \\ I_2(x) &= I_2(\varepsilon) \end{aligned}\right\} \text{for } x \geqq \varepsilon$$

Eq. (32) may be written in the form

$$v' = \frac{1}{\sigma U}\sqrt{I_1^2(\varepsilon) + I_2^2(\varepsilon)}\, e^{-\frac{k}{2U}x} \sin(\sigma x - \delta) \tag{34}$$

where

$$\tan \delta = \frac{I_2(\varepsilon)}{I_1(\varepsilon)}.$$

Thus the distribution of the meridional velocity component outside the region where the external force is acting is given by a damped harmonic curve whose wave length and damping coefficient are equal to the wave length and damping coefficient of the free stationary wave.

To show this by a concrete example it will be assumed that the external force has a constant finite value K in a narrow strip of width ε and that it vanishes outside this strip, that is

$$\begin{aligned} K(x) &= K \text{ for } 0 \leqq x \leqq \varepsilon \\ K(x) &= 0 \text{ for } x < 0 \text{ and for } x > \varepsilon \end{aligned} \tag{35}$$

Other more elaborate functions might represent the distribution of the solenoids better. Such refinements would give more complicated expressions for v'. But they do not appear necessary at present since detailed observations of these disturbances are not yet available.

With the expression (35) for the external force (32) becomes

$$v' = \frac{K}{\beta}\left[1 - \frac{1}{\sigma}\sqrt{\frac{\beta}{U}}e^{-\frac{k}{2U}x}\cos(\sigma x - \gamma)\right] \qquad \text{if } 0 \leqq x \leqq \varepsilon \tag{36a}$$

$$v' = \frac{K}{\beta}\frac{1}{\sigma}\sqrt{\frac{\beta}{U}}\, e^{-\frac{k}{2U}x}\left[e^{\frac{k\varepsilon}{2U}}\cos[\sigma(x-\varepsilon)-\gamma] - \cos(\sigma x - \gamma)\right] \text{ if } x \geqq \varepsilon \tag{36b}$$

where

$$\tan \gamma = \frac{k}{2\sigma U}.$$

The perturbation velocity v' and the perturbation vorticity $\frac{dv'}{dx}$ are continuous for $x = \varepsilon$ and vanish when $x = 0$.. In the limiting case when the width

ε of the strip in which the external force acts tends to zero it follows from (36b) that

$$v' = \frac{\varepsilon K}{\sigma U} e^{-\frac{k}{2U}x} \sin \sigma x. \tag{37}$$

Thus, as ε tends to zero K must tend to infinity in order that $\varepsilon \cdot K$ may remain finite and so give a finite value for v'. As long as ε is small the forced perturbation is represented by a damped sine wave.

When ε becomes comparable to the stationary wave length $2\pi/\sigma$ the meridional velocity component v' may be computed from (36a) and (36b). To give an example it has been assumed that $U = 8$ m/sec, $\varepsilon = 1000$ km, $\varphi = 45°$, $k = 5 \cdot 10^{-6}$ sec^{-1} and $K = 10\beta$. The velocity has been calculated for two cases:

a) disregarding the influence of friction,
b) taking the influence of friction into account.

The results are represented in Fig. 4. When the effect of friction is neg-

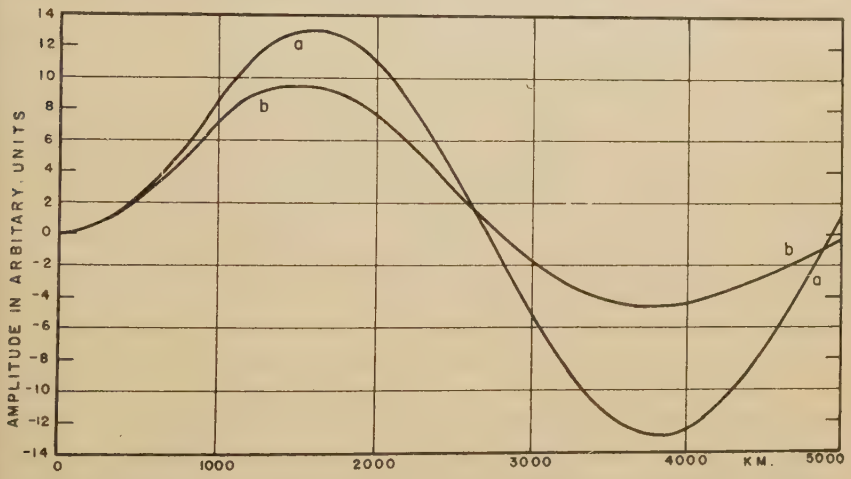

Figure 4. Effect of a concentration of solenoids on the intensity distribution of a stationary perturbation; (a) without friction (b) under the influence of friction.

lected (36a) and (36b) may be changed by a simple trigonometric transformation to

$$v' = 2\frac{K}{\beta} \sin^2\left(\sqrt{\frac{\beta}{U}}\, x\right) \qquad \text{for } 0 \leqq x \leqq \varepsilon \tag{38a}$$

$$v' = 2\frac{K}{\beta} \sin\left(\sqrt{\frac{\beta}{U}}\,\frac{\varepsilon}{2}\right) \sin\left[\sqrt{\frac{\beta}{U}}\left(x - \frac{\varepsilon}{2}\right)\right] \qquad \text{for } x \geqq \varepsilon \tag{38b}$$

These equations show that when no friction is acting v' is proportional to the square of a sine function in the region where the external force acts and proportional to a sine function outside this region. When the frictional forces are effective the conditions are similar as a comparison of the curves (a) and (b) of Fig. 4 will show. In particular, the disturbance has the form of a damped harmonic curve outside the region where the external force is acting.

Over these stationary forced disturbances free migrating disturbances may be superimposed. The appearance of the resultant disturbances may therefore become rather complicated. When five-day mean charts are plotted (*3*), however, the effect of the non-stationary components is partly smoothed out so that the pattern of the charts becomes simpler and resembles more closely the one produced by the external solenoidal forces

SUMMARY

In this paper an investigation by Rossby concerning the effect of the latitudinal variation of the Coriolis force on the propagation of atmospheric disturbances is extended to allow for the finite lateral extent of the disturbances, for the influence of friction and for the action of an external force. When the lateral extent of the disturbance is finite, the difference between geostrophic wind and velocity of propagation of the disturbance becomes smaller than in Rossby's case. When the disturbance is stationary its wave length is larger than in the case of infinite extent.

The main effect of friction is to damp the disturbance. The external force which is assumed to be a function of the longitude generates a stationary disturbance of the same wave length as the free disturbance would have if there were no external force.

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A DETAILED STUDY OF THE SURFACE LAYERS OF THE OCEAN IN THE NEIGHBORHOOD OF THE GULF STREAM WITH THE AID OF RAPID MEASURING HYDROGRAPHIC INSTRUMENTS

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INTRODUCTION

In an earlier communication in this Journal the writer reported on a Bathythermograph for obtaining continuous records of temperature against depth in the first 150 meters of the ocean (Spilhaus 1937). While further development of this preliminary instrument revealed it to be rather critical to vibration, the results obtained with it, some of which were reported in the paper cited above, were sufficient to indicate the desirability of further studies and development along these lines. Abrupt changes in the vertical temperature distribution with very small changes in horizontal distance together with extremely complex temperature structure in the vertical were observed in the surface layers. When an attempt was made to reproduce these instruments, various devices were employed in order to shield the bi-metal reed, which was then used as the temperature element, against the vibrational forces as it travelled through the sea water. While these could be reduced to a large extent when the instrument was used under favorable circumstances, it was not found possible to completely eliminate vibrations under all conditions because the bi-metal temperature strip formed an integral part of the recording stylus and had, necessarily, in order to reduce the transient errors in the record when the instrument passed rapidly through a thermoclime, to be exposed to the turbulent flow of the water through the instrument to insure adequate heat transfer to the temperature element. This incompatibility of the requirements was eliminated by the use of the bourdon type element where the bulb of the temperature responsive system may be exposed to the flow while the moving portion, the actual bourdon spiral, may be entirely shielded from flow. This is in essence the basic change made in the new Bathythermograph.

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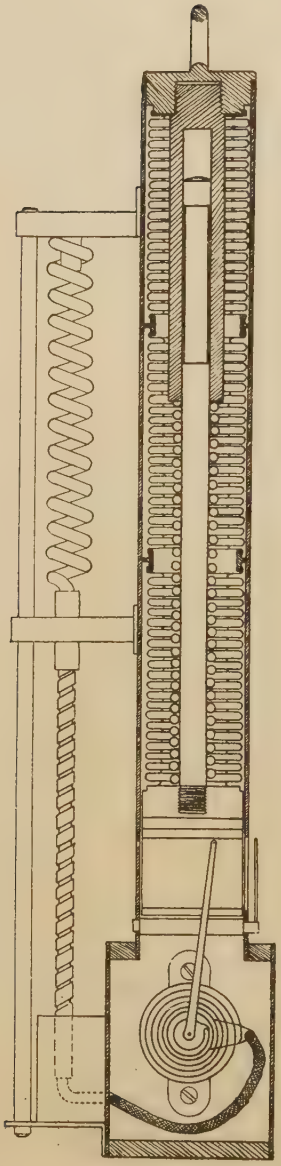

Figure 5. Diagram of Bourdon-type Bathythermograph.

DESCRIPTION OF THE BOURDON TYPE BATHYTHERMOGRAPH

The new Bathythermograph, incorporating the bourdon element described above, is shown in Figure 5. It will be seen that the instrument consists of a pressure responsive element which carries a holder into which a small glass slide may be slipped while the temperature responsive unit is fixed in the body of the instrument so that the slide, which receives the record, is moved parallel to the length of the instrument by pressure changes while temperature changes move the pen across the slide. Thus a plot of temperature against pressure is obtained on the glass. The slide is prepared for receiving the record in an identical fashion to that described by the writer (Spilhaus 1937). Figure 6 shows a photograph of the instrument, together with a box of records, the latter indicating the convenience of the small slides, for, when measurements are made in rapid succession, many hundreds of records may be taken on a single cruise. It may be noticed in the photograph that a Nansen bottle clamp is used to fasten the Bathythermograph to the hydrographic winch wire. This clamp with its tripping mechanism is used also to provide a means whereby, when the instrument reaches the lowest point of the descent into the ocean, a messenger may be sent down the hydrographic wire in the usual fashion to take off the pen, preparatory to the ascent of the instrument, so that only one trace, that of the descent, is obtained. In the case of the Bathythermograph soundings when the vesssel is not moving relative to the water and when the tem-

perature conditions are steady there is considerable value of having both traces and insuring that the two traces correspond as this is an excellent check as to whether the instrument is working perfectly. However, where the instrument was used while under way at sea, it was found that the descent and ascent curve of the instrument might differ quite considerably, not due to imperfect functioning of the instrument but due to the fact that actual

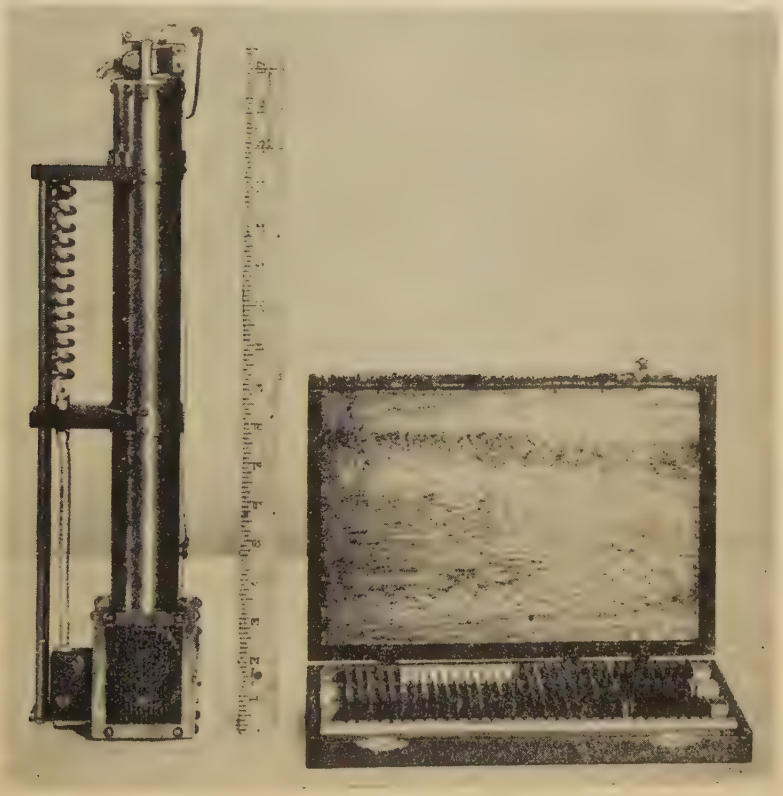

Figure 6. The Bourdon-type Bathythermograph and box of record slides.

horizontal gradients of temperature may be so great in the surface layers that considerable changes of temperature, level for level, are found due to the distance the ship has travelled in the interval between the time the instrument passed the particular level in its descent and the later time when it passed it again during the ascent. This is clearly brought out by examination of the later diagrams. The transient rate of the temperature element was found to be very rapid and certainly sufficient for all practical purposes. This high rate

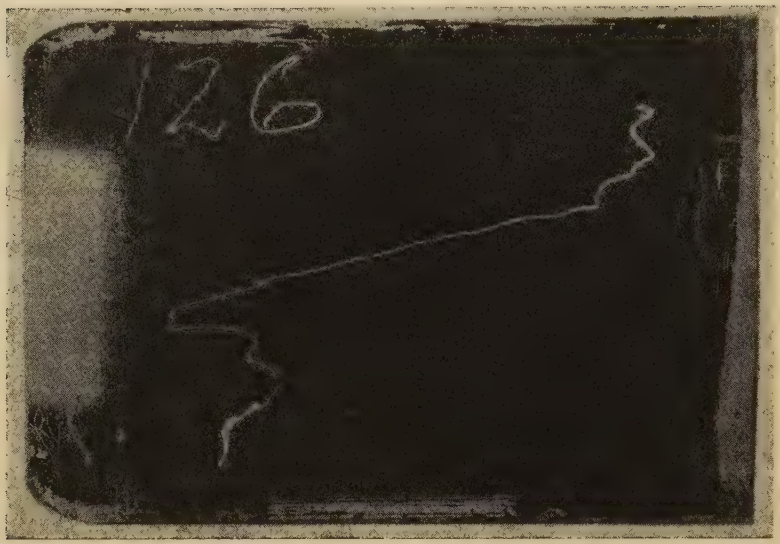

Figure 7. Enlargement of original record from Bathythermograph station No. 126.

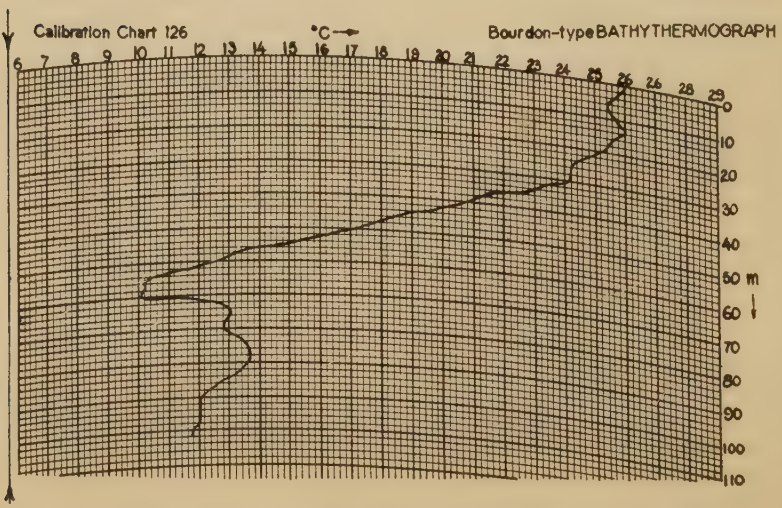

Figure 8. Same record as in figure 7 superimposed upon calibration chart.

of response was obtained by having the bulb constructed in helical form of a tube of small diameter, thereby obtaining a high surface-volume ratio. A typical trace is shown in Figure 7. Figure 8 shows the same trace superimposed on a calibration chart. The rise of temperature of about 3.0° Centigrade between 70 and 73 m. is noteworthy. The instrument was descending at that time at a rate of 20 meters per minute. In other words, this response took place in less than 10 seconds. Similar sharp inversions are observed very often on other records. The method of using the traces obtained on the small glass slides is very simple: a special projector throws an enlarged image of the trace on a frosted glass screen on which may be placed a translucent calibration chart. The graph of temperature against depth is then simply traced with pen or pencil onto the calibration chart. The slide holder in the projector is arranged to be in every respect similar to the holder in the Bathythermograph and, therefore, the slide is held in the same relative position in each. Thus, there is no need of any reference marks on the slide.

So completely did the use of the bourdon thermometer eliminate vibration that it was found the new Bathythermograph could be sent down into the ocean while the vessel was travelling at as high a speed as 11 knots. In fact, not the slightest vestige of vibration was apparent on a record made at this speed, the only difficulty experienced being that a considerable length of wire has to be paid out to get down to the requisite depth. This ability to use the Bathythermograph while travelling at high speed was very shortly recognized as a tremendous advantage because by this means a closer approach to synchronism of observations was made possible.

It was realized the value of the temperature depth soundings taken while under way would be greatly increased if it were possible to develop an instrument for taking salinities at the same time. The ideal solution, of course, would be the development of a recording salinity instrument whereby a continuous record of salinity against depth could be taken, preferably while the ship was in motion. However, the development of such an instrument, while entirely possible, will take some considerable time and it was felt that it is sufficient to have a continuous record of temperature and measurements of salinity at suitably distributed depths. This led to the development of a Sea Sampler for taking samples of the water at various predetermined depths while under way.

DESCRIPTION OF THE PRESSURE-OPERATED MULTIPLE SEA SAMPLER

The requirements to be met in the design of the Sea Sampler were, that wire angle must have no effect on the operation, that the instrument must take a number of samples at different depths and that the whole instrument should not be too bulky and clumsy in operation. The Pressure-operated

Multiple Sea Sampler comprising six sample bottles arranged such that they tripped at a distribution of points down to 150 meters was developed, and the experimental model was built for the writer through the kind cooperation of the Submarine Signal Company of Boston, Massachusetts.

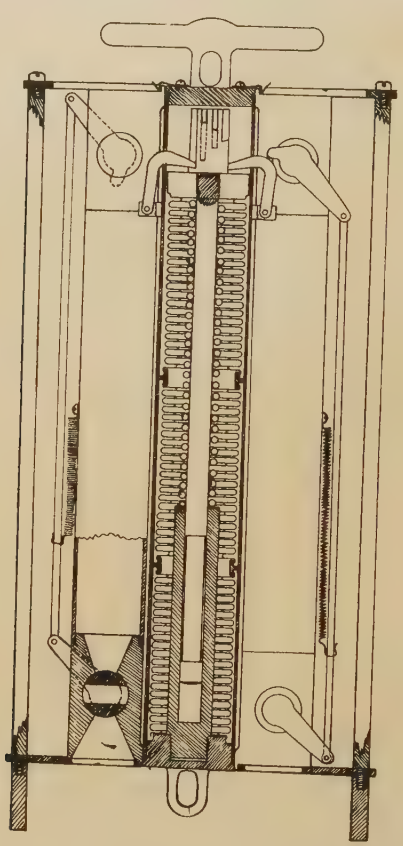

Figure 9. Diagram of Pressure-operated Multiple Sea Sampler.

A diagram showing the mechanism of the instrument is given in Figure 9 and a photograph of the completed instrument together with its carrying case and with two of the water bottles removed, is reproduced in Figure 10. Examination of these figures will show that essentially the instrument consists of a pressure responsive unit exactly similar to that used in the Bathythermograph which carries on its moving end a release controller which consists of a barrel with slots cut parallel to its axis. These slots are of dif-

ferent lengths and thus the trigger which presses on the barrel for each water bottle reaches the slot at a different pressure. Thus, each water bottle is released at a pressure which is determined by the length of the slot. Actually, in the instrument as built by the Submarine Signal Company, the release

Figure 10. The Pressure-operated Multiple Sea Sampler, showing two of the water bottles removed, and the carrying case.

controller is so constructed that the triggering points of the different bottles may be varied at will, so that a distribution of depths for the six water bottles may be chosen according to the particular data required in each case.

It should be possible to use the Pressure-operated Sea Sampler without stopping the vessel, although its bulk means that in all probability practical considerations will limit the speed of the vessel at which it can be lowered because its considerable drag will make it necessary to pay out a great length of wire in order to obtain the requisite depth. With the present Sea

Sampler, salinities have not been obtained while under way; however, this will be attempted directly the "Atlantis" is equipped with a hydrographic winch wire of sufficient strength so that there will be no danger of losing the instrument. In view of the fact that only one experimental model has been made, it has not been deemed advisable to risk the instrument on the regular hydrographic wire while the vessel is moving until a spare Sampler is available.

A considerable number of tests have been made with the Sea Sampler and these in general show slight differences in the Sea Sampler salinities from

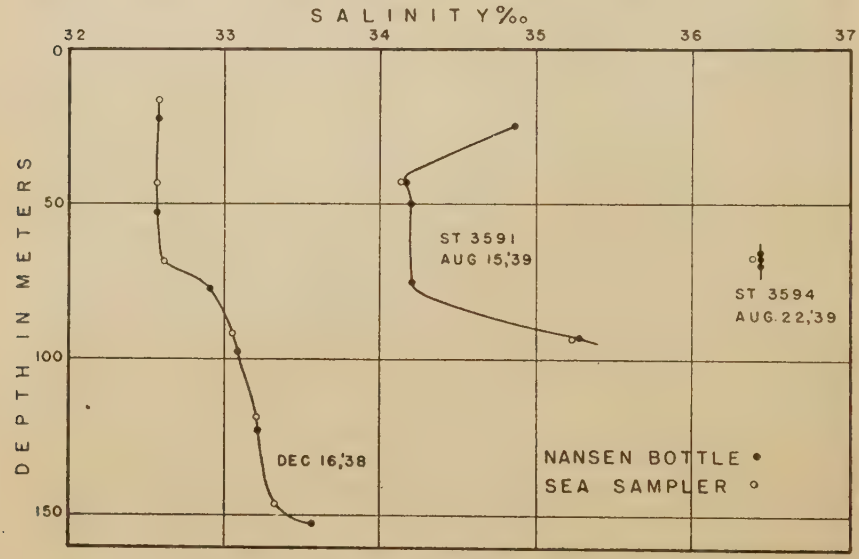

Figure 11. Comparisons between salinities obtained from Nansen Bottle samples and Sea Sampler specimens.

those obtained by the regular Nansen bottle technique. However, in many cases the differences may not immediately be ascribed to improper functioning of the Sampler, as it is extremely difficult to insure that the Nansen bottle trips at exactly the depth for which the Sea Sampler is set and it is, furthermore, extremely difficult to arrange that the two instruments take their samples at exactly the same time. Therefore, the differences observed in the tests may be due to the fact that the samples taken with the Nansen bottle were either taken not at the same depth or not at the same time, and both these factors may easily introduce real differences in the salinities of the order of magnitude found in the tests. In view of the fact that the Sampler is designed for use in the surface layers of the ocean where the salinity

variations are considerable, greater errors in the salinities can be tolerated than would be possible at greater depths. Figure 11 shows curves of salinity against depth for some of the comparisons made at various times. While the differences indicated are not smaller than the usually recognized allowable error, they are small enough so that pending further development of the Sampler the instrument is useful.

Tests made in the laboratory to determine the triggering points of the various sample bottles reveal a high degree of accuracy as far as the repetition of the release pressure is concerned. The following figures were obtained in the pressure chamber of the Submarine Signal Company:

Water Jar	*Release Pressure*	*Equiv. Depth*
1	24 lbs. per sq. in.	16.3 meters
2	63 lbs. per sq. in.	42.8 meters
3	101 lbs. per sq. in.	68.7 meters
4	135 lbs. per sq. in.	91.7 meters
5	174 lbs. per sq. in.	118.3 meters
6	215 lbs. per sq. in.	146.2 meters

Repetition of the test indicated that the release pressure never varied more than one pound from the release pressure indicated in the above table; that is, in general, the bottles will trip within plus or minus half a meter of the predetermined depth. Tests at sea in conditions where the wire angle was entirely negligible also revealed a tripping accuracy well within the accuracy of the meter wheel. It is for this reason that confidence may be placed on the fundamental principle of the Pressure-operated Sea Sampler but it is realized that further development is required. Particularly is investigation required of the degree to which throttling at the stopcocks of the sample bottles is effective in slowing up the exchange of water within the bottles and thereby causing a lag effect on the indicated salinities. For the purpose of studying this throttling effect, two small water bottles were constructed such that the body of the bottles was of the same cross section as the opening in the stopcock. It was thought that this would reduce the throttling effect. It is surprising, however, to note that all samples obtained by means of these smaller bottles were in far greater error than those obtained with the large bottles, and this would indicate that the erroneous readings are due to contamination of the sample rather than the throttling effect, because such contamination, whether it be due to leakage at the stopcocks or the introduction of water of different salinity by some other means, will have a greater effect the smaller the sample taken.

RESULTS OF TWO CRUISES WITH THE RAPID MEASURING HYDROGRAPHIC INSTRUMENTS

During the summer of 1939 two cruises of the "Atlantis" were carried out on which the instruments described above were used. On the first cruise in July, the Sea Sampler did not operate with sufficient regularity to permit it to be used for actually obtaining useful data. The functioning was impaired by the fact that spring tension to bring the trigger out again after its depression into the release controller was insufficient and on most bottles the triggers were thus broken off during the ascent of the Sampler. As no facilities for the repairing of these triggers were available on board the instrument could not be used on the remainder of the cruise.

The Bathythermograph operated extremely satisfactorily and a continuous section for a distance of 175 miles was made with a single break of 30 miles in which no records were taken. During this stretch of 175 miles, 51 records were taken, that is, with the exception of the 30 mile break, a record on an average for every three miles. Most of the time during this section the "Atlantis" was travelling at a speed of about 7 knots, although on certain occasions the speed increased to as high as 11 knots. No trouble was experienced in using the Bathythermograph at such increased speeds beyond the necessity of paying out more wire in order to counteract the increase of wire angle. The procedure adopted in working a continuous section with the Bathythermograph is to lower the weight, about 120 lbs. of lead, into the water and keep it in the water, then to attach the Bathythermograph to the hydrographic wire at deck level so that the insertion and removal of slides may be accomplished without removing the instrument from the wire. After the slide is inserted the Bathythermograph is lowered at full speed into the water. Approximately 400 meters of wire must be paid out in order that the instrument may descend to 120 meters depth when the vessel is travelling at 7 knots.

The course of both cruises on which the Bathythermograph was used is shown on Figure 12. The line entered between Atlantis station No. 3562 and continuing beyond Atlantis station No. 3565, indicates the position of the first Bathythermograph section. The continuous section is represented in Figure 13 in the form of a diagram showing the isotherms down to a depth of 120 meters for a portion of this course.* Only 60 miles of the section is shown in Figure 13. The remaining parts of the section indicated a very regular horizontal distribution of the isotherms in both the tests made before Station 6 and those made after Station 28, so that they have not been included in the figure. The distribution of the isotherms, however, in Figure 13 reveals features of extreme interest; the boundary of the Gulf Stream proper is situated approximately at Station 11 on this diagram and

* c. f. Fig. 12 for position of the portion of the course analyzed in Fig. 13.

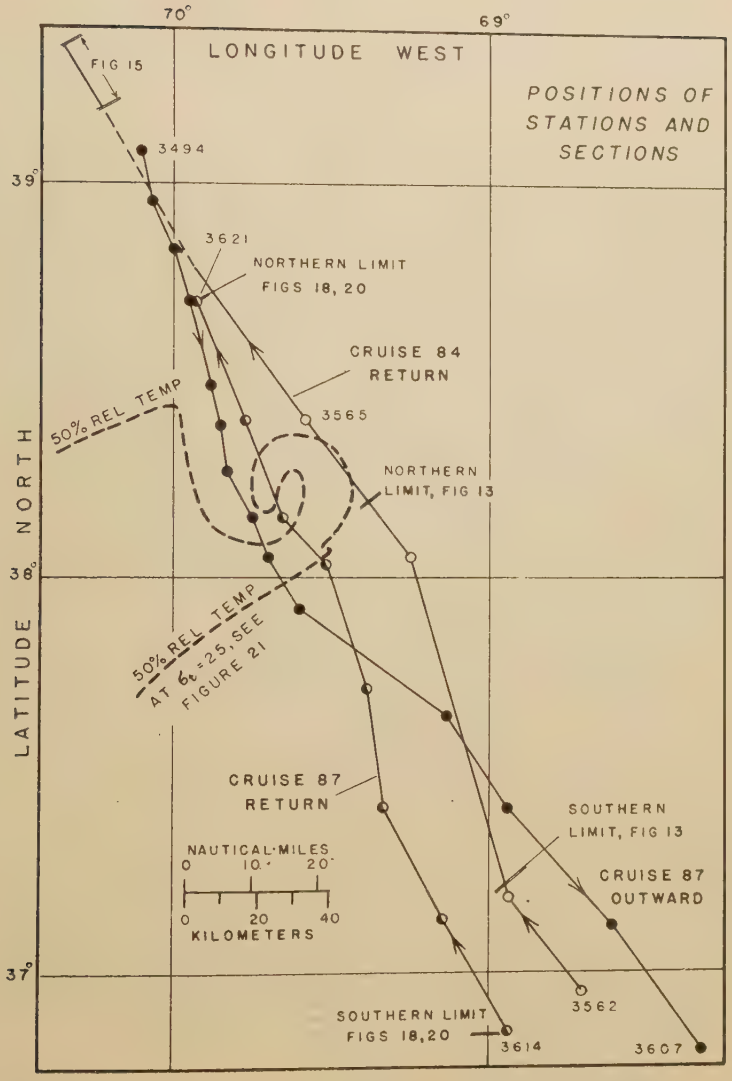

Figure 12. Chart showing positions of stations and sections. On the return route of cruise 84, full line indicates continuous Bathythermograph operation. Routine hydrographic stations marked by open circles. On the outward route of cruise 87 only routine hydrographic stations were made, as indicated by black dots. On the return route of cruise 87, full line indicates continuous operation of Bathythermograph, with Multiple Sea Sampler stations shown by black and white semicircles. Broken line indicates approximate course of 50% relative temperature boundary as shown in Figure 21.

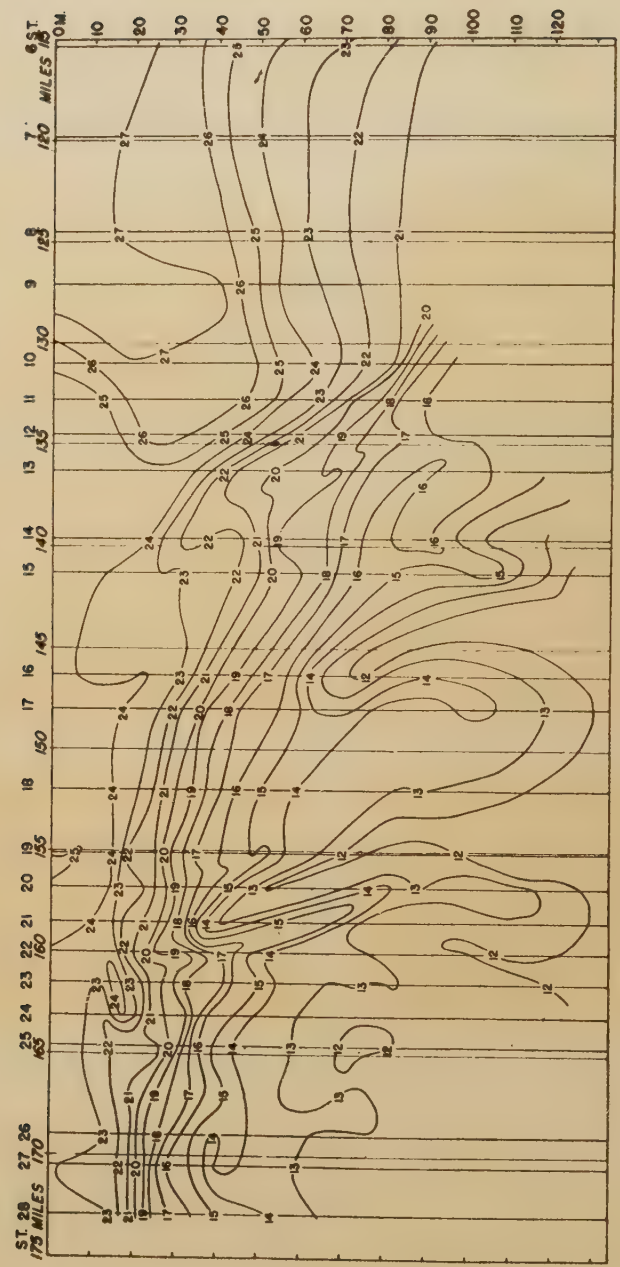

Figure 13. Distribution of isotherms in the surface layer in the neighborhood of the northwestern edge of the Gulf Stream (Cruise No. 84, July 1939).

it is in the water to the left of this point (looking down stream) that the isotherms are most disturbed. The 14° isotherm, for instance, exhibits in a 12 mile horizontal distance, between Stations 16 and 21, a variation from a depth of 65 meters at Station 16 to 110 meters at Station 17 and rising

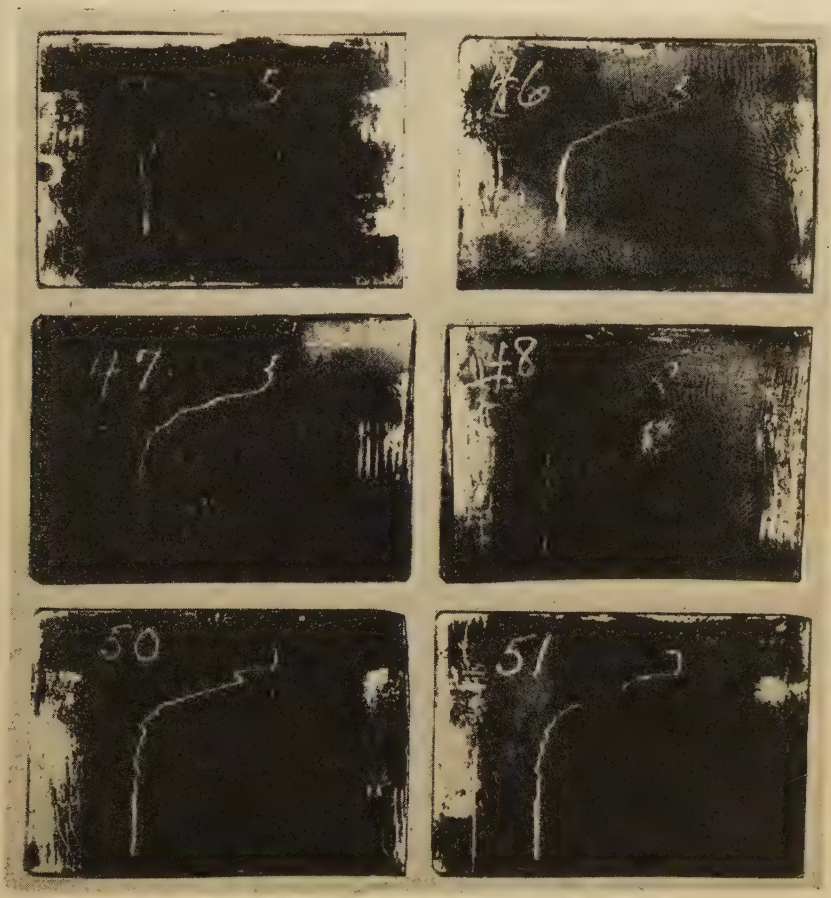

Figure 14. Some of the original records from which the data for figure 15 were obtained.

to a depth of only 40 meters in Station 21. These perturbations suggest the presence of a small eddy of the order of magnitude of 10 to 15 miles in size. Furthermore, from a knowledge of the general configuration of the isopycnals in these layers at the edge of the stream as given, for instance, in Figure 26 of Iselin's "Study of the Circulation of the Western North Atlantic,"

(Iselin 1936) it would seem that the shallower points of the 14° isotherm would lie along one ispycnal while the deeper points would fall along a different isopycnal, indicating that the water at the various shallower points was of the same origin (slope water) and that of the various deeper points of a different origin (Gulf Stream water). Because no salinities were taken simultaneously with the Bathythermograph soundings, it is not possible to

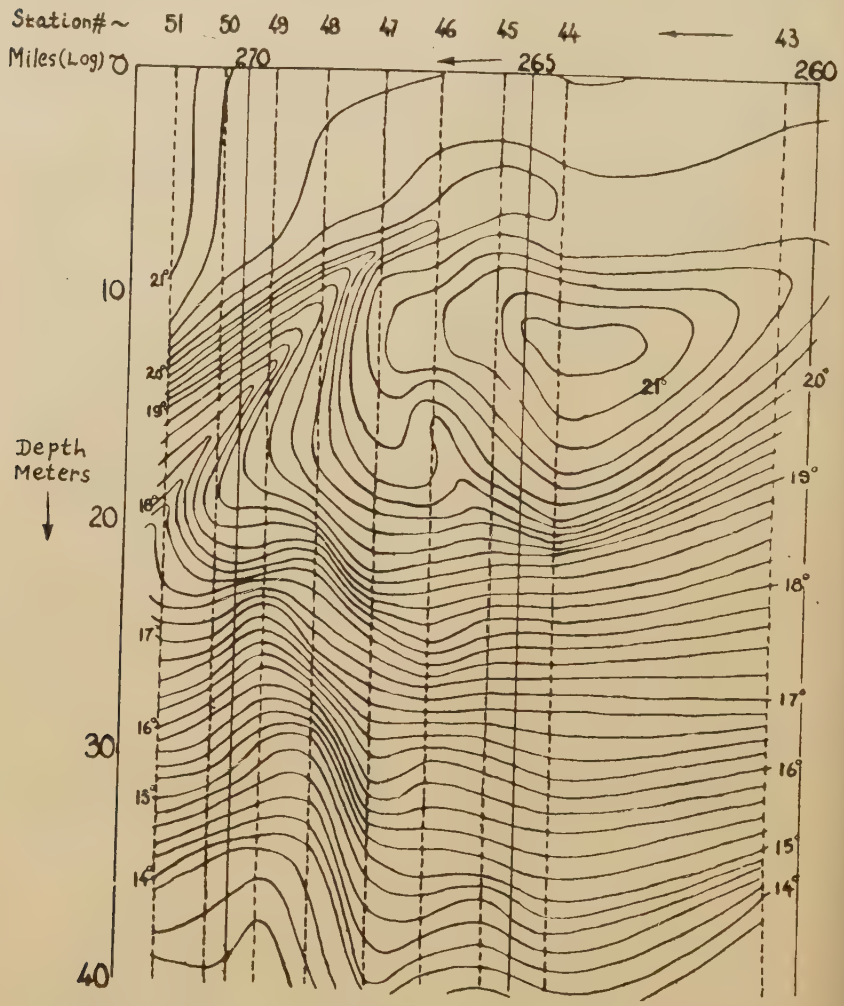

Figure 15. Distribution of isotherms in a short section (see Fig. 12), showing in detail the structure of a discontinuity zone in the surface layers.

establish the positions of the isopycnals in this particular case. On the second cruise densities were taken with the Sea Sampler and the above conclusion was ratified. This will be shown later.

A small section extending for a distance of 10 miles was made 30 miles to the north-west of the end of the continuous Bathythermograph section described above; the position of this section is shown in Figure 12 starting at Long. W. 70° 15′ and Lat. N. 39° 15′ approximately, and in this case Bathythermograph slide, No. 43, indicated a surface inversion of temperature starting at a depth of approximately 10 meters the highest temperature being recorded at about 13 meters. Similar surface inversions have been observed before, but not adequately explained. Therefore, directly this inversion was noted the Bathythermograph was operated rapidly and continuously so that for a 7 mile stretch records were obtained at intervals of less than a mile. Thus a very detailed analysis of this surface inversion could be made. The records are reproduced in their original form in Figure 14 and the inversion under scrutiny is that which occurs in the layer immediately over the summer thermocline. The nature of the inversion becomes clear when a detailed diagram of the isotherms in the first 40 meters is plotted. This diagram is shown in Figure 15, and it is likely that the inversion marks a sloping zone of discontinuity between the two water masses of slightly different characteristics. If, as is probable, although it cannot be proved in the present case as no salinities were taken, the sloping zone of discontinuity is bounded by surfaces of constant density, then Figure 15 provides an interesting pictorial representation of a "Margules slope" in the ocean.

Results of the Second Cruise.—After the Sea Sampler had been repaired and improvements incorporated to ensure its satisfactory operation, a second cruise was undertaken on the "Atlantis" with these instruments in August, 1939. The routine part of the cruise was to make one of the regular Gulf Stream sections as part of the Woods Hole program in connection with the Gulf Stream investigation. It was arranged, however, that on the return course of the vessel no stops would be made so that a continuous semi-synchronous section could be obtained with the Bathythermograph once more. Furthermore, provision was made to take water samples with the Pressure-operated Multiple Sea Sampler at intervals spaced further apart than the Bathythermograph stations but nevertheless close enough so that the distribution of isopycnals could be obtained. In Figure 12 the course of the second cruise is shown, and it is seen that the section is not far removed from the previous one and so it was to be expected that, if the small scale eddy observed at the edge of the Gulf Stream on the previous cruise and indicated by the disturbance of the isotherms in Figure 13 were real, it would be repeated in this case.

Figure 16 shows a plot of the isotherms for the cross section made by

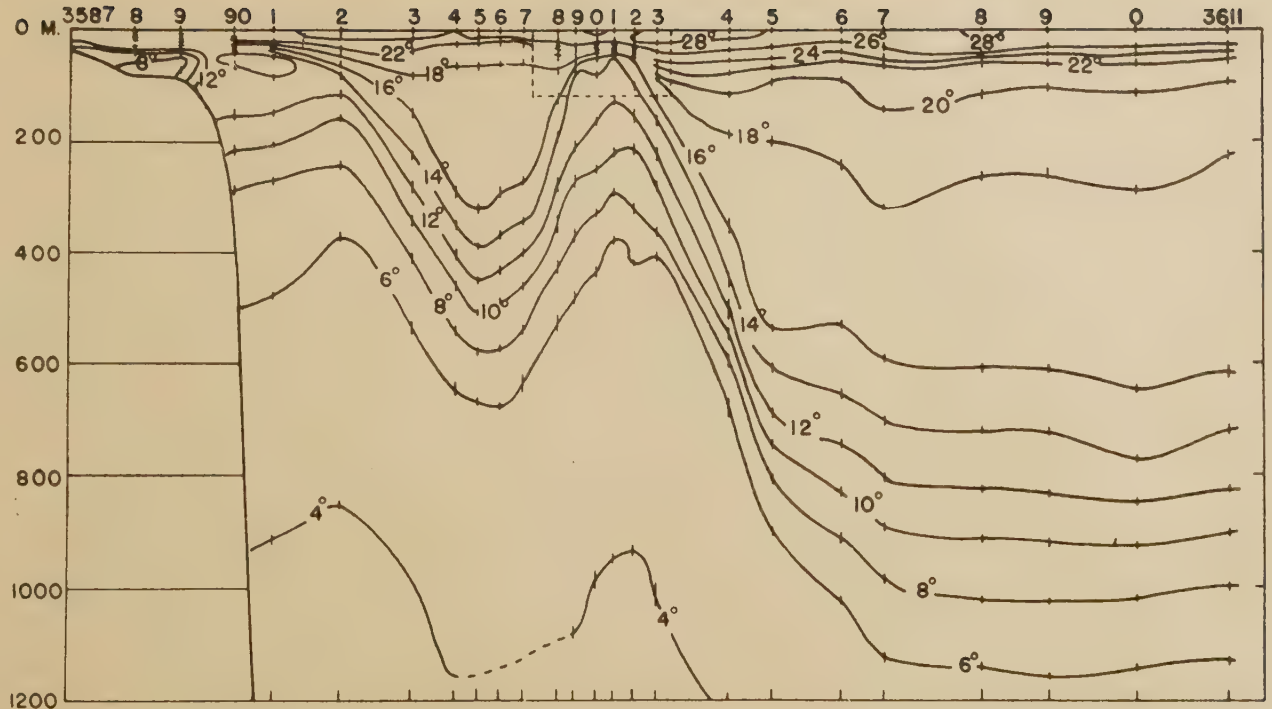

Figure 16. Distribution of the isotherms as determined by reversing thermometers on the Montauk Point to Bermuda section. Relative position of Bathythermograph section given in Figure 17 shown by rectangle in broken lines between stations 3597 and 3604 (Cruise No. 87, August 1939).

means of the regular hydrographic technique on the outgoing course of the vessel. An extremely well developed, large eddy is present on this occasion and this would imply that there is a high rate of shear and therefore an even better chance for the development of small scale eddies in the neighborhood of the edge of the Stream. The position of the Bathythermograph section is indicated by the dotted rectangle extending horizontally from station 3597 past station 3603 and down to a depth of approximately 120 meters. This section is shown in the detail revealed by the Bathythermograph analysis made on the return in Figure 17, and a similar temperature structure to that obtained in the previous Bathythermograph section is again found. Furthermore, just as in the case of the section shown in Figure 13, the shallower positions of any one isotherm appear to lie along a line which would correspond to an isopycnal and the deeper positions along another ispycnal. However, in this case, with the aid of the salinities obtained by the Pressure-operated Multiple Sea Sampler, the actual isopycnals may be plotted and these are shown in Figure 18.

The distribution of the isopycnals obtained by the new rapid measuring instruments and shown in Figure 18, may be compared with those obtained on the outgoing section by the regular hydrographic technique and shown in Figure 19. The general configuration of the isopycnals is identical although the absolute depth at which any one density occurs is not the same on the two occasions. It is entirely possible that such changes could occur in the intervening few days between the outgoing stations in this region and the incoming ones.

The spacing of the stations at which samples were taken for salinity determinations and, therefore, for which the depth of the isopycnals may be computed was not as close as the spacing of the Bathythermograph stations. It therefore might seem that in drawing the course of the isopycnals in Figure 18 and also in Figure 19, the latter from densities obtained by the regular hydrographic technique, considerable smoothing has been carried out and that actually the isopycnals may wander in between the points at which salinity was determined. Unless very short intervals are made between the Sea Sampler stations, no definite evidence to prove conclusively that the isopycnals do not wander can be obtained. In a recent study (Seiwell 1939) the assumption has been made that the changes in depths of the isotherms are entirely due to changes in depth of the isopycnals, where here the assumption made is exactly the reverse, in other words, that changes in depth of the isotherms are due entirely to changes in the temperature salinity correlation. It is probable that neither of these extremes is entirely permissible. Seiwell's analysis, however, was for much deeper layers than the present one and indirect evidence in our case would indicate that the assumption made here is valid. For instance, the comparison of Figure 18 with Figure 19 which reveals an identical general configuration of the iso-

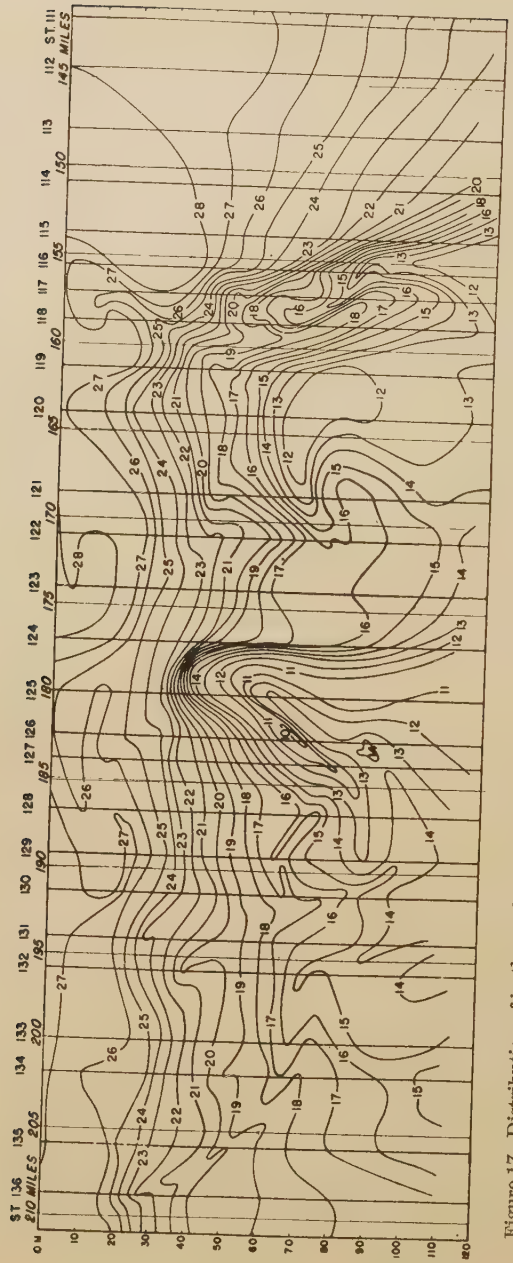

Figure 17. Distribution of isotherms in the surface layers in the neighborhood of the northwestern edge of the Gulf Stream as shown by Bathythermograph records (Cruise No. 87, August 1939).

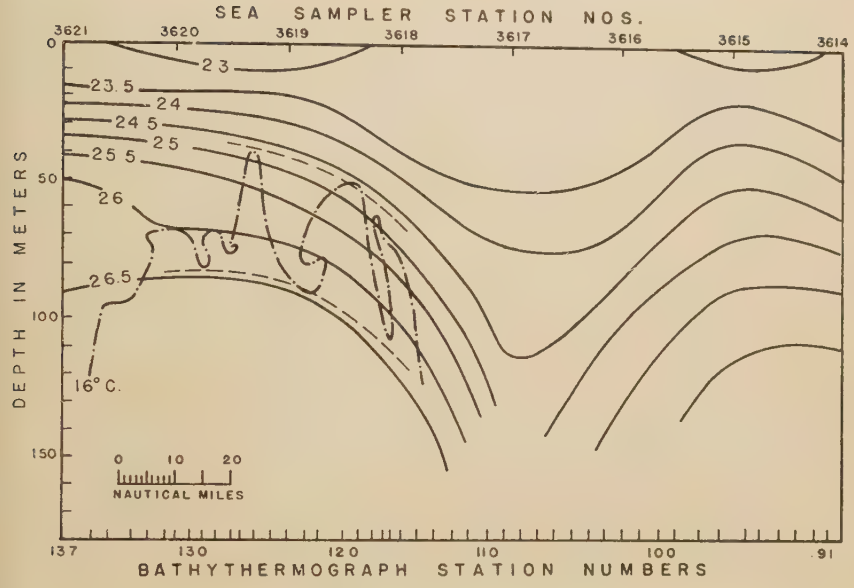

Figure 18. Distribution of isopycnals and 16° isotherm from Bathythermograph and Sea Sampler data.

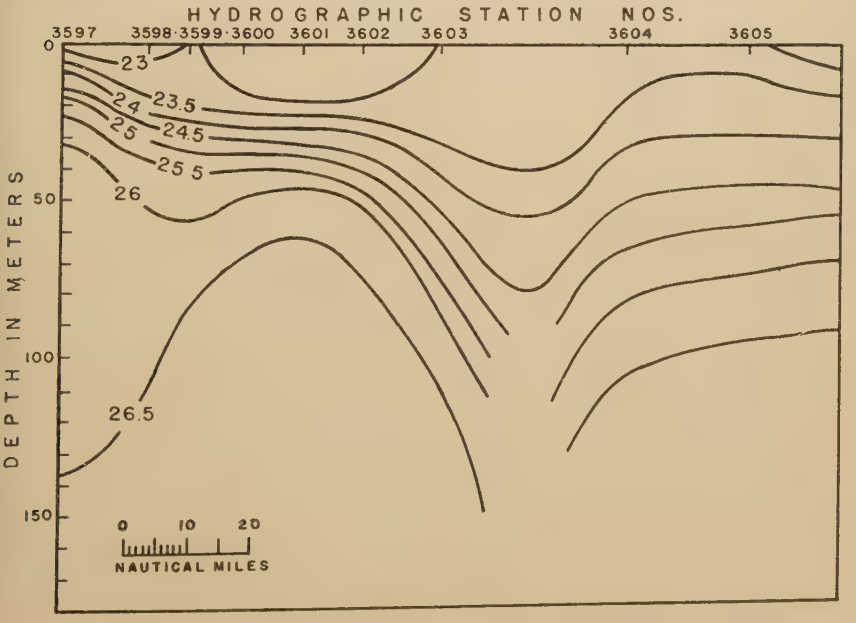

Figure 19. Distribution of isopycnals according to observations from routine hydrographic stations.

pycnals on the outgoing and incoming parts of the cruise must be accepted as rather good evidence that there is no small scale distortion of the isopycnals, especially in view of the fact that on the outgoing part of the cruise, regular hydrographic stations were made at extremely short intervals; the part of this section, which corresponds to that region where the complex temperature structure is observed on the return part of the cruise, lies between "Atlantis" stations 3598 and 3603 and here six hydrographic stations were made in a distance of about 50 miles. Even with the stations as close as this the isopycnals are very smooth and regular (c. f. Figure 19). Furthermore, if the data is examined from sea sampler stations 3618 and 3619 (corresponding Bathythermograph station 116 and 123 respectively) by comparing the salinity at the 15° Isotherm for example, the following result is found:—

Bathythermograph Station No.	*Sea Sampler Station No.*	*Salinity °/₀₀ at the Depth of the 15° C. Isotherm*	*Depth of the 15° C. Isotherm Meters*
116	3618	34.60	77
123	3619	35.97	102

This shows an extremely marked change in the T-S correlation which is in agreement with our assumption of a relatively smooth distribution of density.

In order to test the suggestion made above regarding the relation of the isotherms to the isopycnal surfaces the 16° isotherm has been sketched in on Figure 19. It is immediately seen that the wanderings of the isotherm in the vertical occur definitely in a layer enclosed by two surfaces of constant density. The 16° isotherm is chosen arbitrarily as an example, though others for other values of the temperature will reveal the same feature. It has been frequently emphasized in oceanography that reliance on the temperature salinity correlation is not justified in the surface layers of the ocean. While it is quite true that there will be departures from the temperature salinity correlation due to surface heating and evaporation effects, in the opinion of the author it is in all probability the presence of small eddies and the consequent lateral mixing which principally upset temperature salinity correlations in the surface layers. Therefore, a method involving temperature salinity correlation will reveal the presence of these eddies if the station spacing is less than the dimensions of the eddy. An analysis was made following the suggestion of Professor A. E. Parr (Parr 1938) to use relative values of the identifying property along isopycnal surfaces. Parr chose to use relative salinity as his identifying value but in view of the fact that in the case of the new rapid measuring technique continuous records of temperature are available it is preferable here to use relative temperature. Obviously, in view of the fact that density is a function of temperature and

salinity, either temperature or salinity on an isopycnal is perfectly suitable for such identification. Therefore, values of the relative temperature obtained in exactly the manner outlined by Parr were worked out and in Figure 20 iso-lines of the 50% relative temperature values have been drawn. These give an indication of the division of the tongues of slope water and tongues of Gulf Stream water, the former with relative temperatures less than 50%, the latter greater than 50%. The figure shows in a

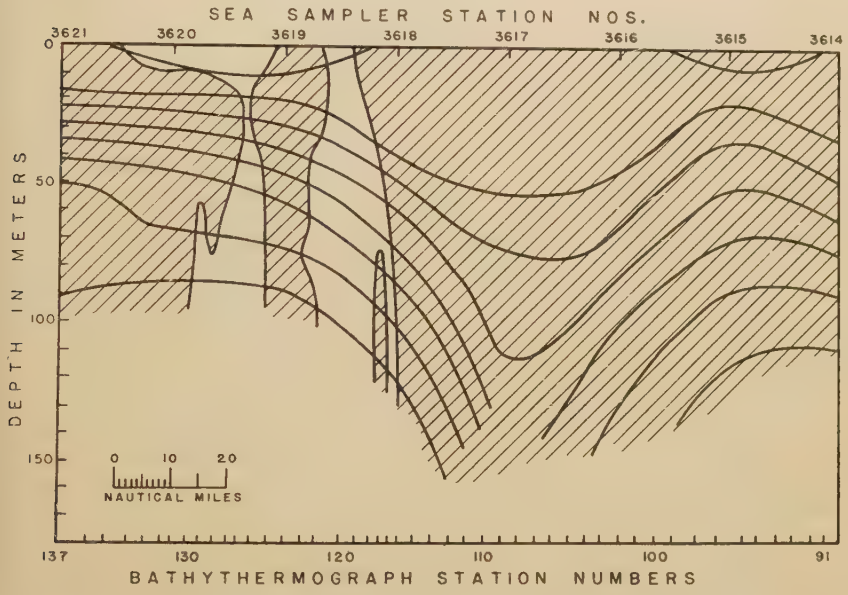

Figure 20. Distribution of relative temperatures in section shown in Figure 18. Relative temperatures above 50% indicated by hatched areas, below 50% by blank regions.

very remarkable fashion that the intrusion of slope water is in the form of a tongue which extends to depths without much lateral displacement.

While explicit conclusions regarding the nature of the eddy (its direction and rotation etc.), which in cross section would give this distribution of relative temperatures, may not be drawn from a single section, it may be shown that a formalized and entirely logical eddy formation will fit the available data. Thus in Figure 21 the temperatures along the surface of $\sigma_T = 25$ are plotted, and a series of isotherms, representing a reasonable eddy formation, are drawn in which satisfy the data. An extremely interesting feature on this diagram is the appearance of a parasite eddy on the right side of the cold tongue. This parasite eddy was indicated on the cross

section on Figure 20. Referring to this figure, it will be seen that there is at Station 117 a small intrusion in warm water for which the 50% relative temperature extends to a depth of 85 meters. Furthermore, on Figure 20 there appears to be a second parasite eddy at Station 129. This second

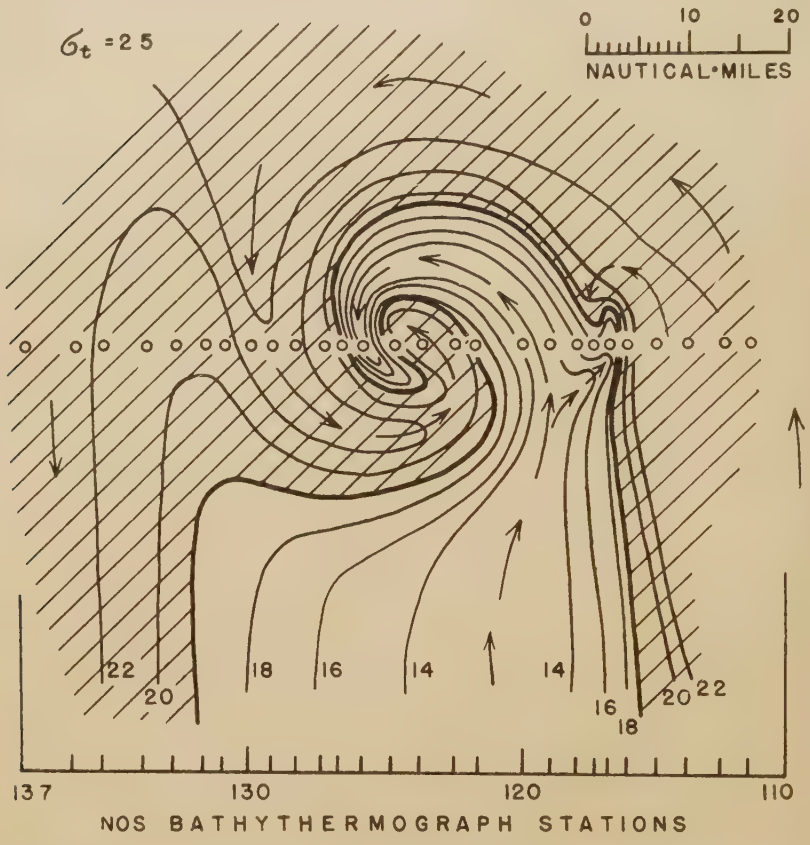

Figure 21. Eddy of intermediate size with parasite drawn to conform with temperature distribution on the isopycnal surface of $\sigma_T = 25$ (somewhat diagrammatical).

parasite on the left side of the cold tongue is not shown up on the surface of $\sigma_T = 25$ but it appears on a similar diagram for $\sigma_T = 26$. Perhaps the appearance of these parasite eddies is one of the most interesting features brought out by this analysis. While they are in no sense incontrovertibly established by the present data and must to a large extent be regarded as speculation on the part of the writer, it is believed that they must exist in

any such eddy formation. Also, the data, such as it is, lends support to the belief in their existence. In all probability, many more parasite eddies would be found on the tongue if a great number of sections were taken so that a well distributed field of temperatures could be plotted on the surface.

It is interesting to observe that in this particular case we have quite clear indications of three distinctly different sizes of eddies. First, the large scale eddy which is shown up by the distortion of the isotherms down to considerable depths as indicated in Figure 16. By the data obtained with the routine hydrographic technique the dimension of this eddy measured as the distance between the axes of the dissimilar tongues, is of the order of magnitude of about 150 kilometers. Secondly, we have the intermediate eddy shown on Figure 21 with a corresponding dimension of about 30 kilometers and, thirdly, the parasite eddies with a dimension of the order of magnitude of 5 kilometers. The intermediate size is of the same order of magnitude as Dr. Rossby's estimate on the basis of a shear of 10^{-5} sec.$^{-1}$ (Rossby 1938), and this would indicate that eddies of these dimensions are numerous enough for their effect on large scale phenomena to be capable of statistical treatment.

PROJECTED OPERATIONS WITH THE RAPID MEASURING INSTRUMENTS

The foregoing preliminary analysis has demonstrated that it is of extreme interest to obtain detailed analyses of the surface layers of the ocean. It is proposed to extend this type of investigation so that either by the cooperation of more than one ship, each equipped with a Bathythermograph and Sea Sampler, or by zig-zagging over the region to be studied, a distribution of points in the isopycnal surface taken semi-synchronously may be obtained. This will enable confirmation of the eddy patterns outlined above to be obtained by an analysis of the type employed by Montgomery (1938).

It is further proposed that after development of the individual mechanisms of the Bathythermograph and Sea Sampler is progressed to a satisfactory point they may be combined such that the same pressure element will be used to trip the Sea Sampler water bottles and also to carry the slide of the Bathythermograph unit. The desirability of making temperature and salinity measurements at exactly the same point even though the position of this point is not so accurately known, has been stressed by Dr. Montgomery and such a combination of the two instruments will insure that the temperature at which salinity sample was taken is accurately known. The proposed combination is illustrated in the diagram Figure 22. It is believed that when each water bottle trips there will be a sufficient jar to the instrument so that the recording stylus of the thermal unit will make

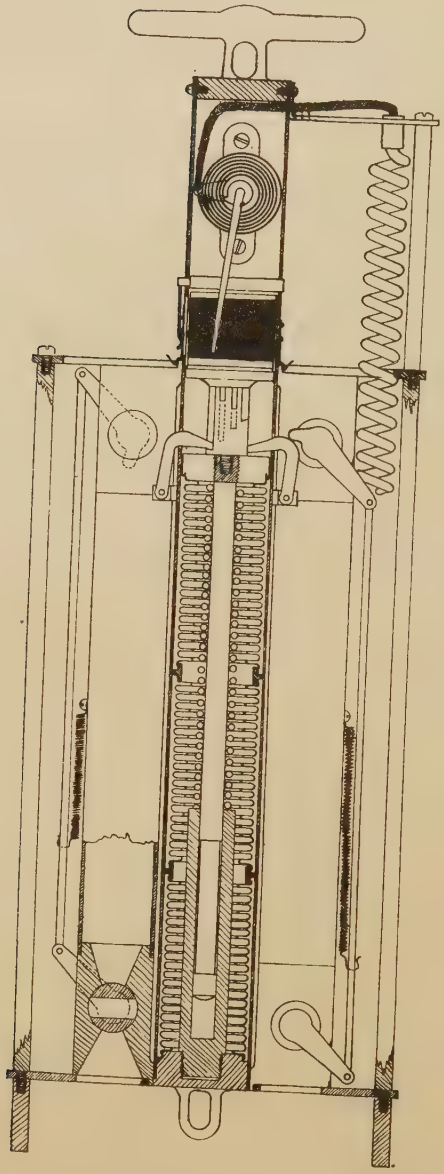

Figure 22. Diagram of proposed combined Bathythermograph and Pressure-operated Sea Sampler.

a noticeable mark on the temperature depth trace. This will be valuable, in addition, for establishing exactly the point of tripping. If it is found that jar caused by the water bottle tripping is insufficient to make such a mark, a simple mechanism is contemplated which will accomplish this.

Such investigations as these, where a complete change in the characteristics of the water may be expected to take place within a distance of a few miles, necessitate fixing the relative positions of the Bathythermograph stations very exactly. In the case of the data described above, the sections plotted for the Bathythermograph are plotted on a horizontal scale representing the reading of the ship's log, that is the distance relative to the water. However, if more than one section is taken the relative positions of the different sections will have to be obtained exactly. If more than one vessel is employed, range finders to obtain relative positions would be very valuable.

In conclusion the writer wishes to express his indebtedness to the Submarine Signal Company of Boston, Massachusetts, who undertook the construction of the instruments described above and without whose assistance the investigation would hardly have been possible.

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VARIATIONS IN THE PHOSPHORUS CONTENT OF ESTUARINE WATERS OF THE CHESAPEAKE BAY NEAR SOLOMONS ISLAND, MARYLAND*

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Studies by Cowles and Brambel (1938), Newcombe and Lang (1939), and Newcombe, Horne and Shepherd (1939) have provided information on the quantitative methods for estimating inorganic phosphorus and, also, on the vertical and horizontal distribution of this nutrient substance in the waters of the Chesapeake Bay. A continuation of distributional work has provided definite information on the magnitude of diurnal, daily and horizontal variations in estuarine waters, and certain points are developed that concern the handling of phosphate samples. Most significant, perhaps, is the evidence pointing to exceedingly rapid utilization and regeneration of inorganic phosphorus under *natural* conditions. It should be pointed out that the quantity of phosphorus present has been measured in the form of inorganic phosphorus (PO_4) only. Were additional data available bearing on the amounts of dissolved organic phosphorus as well as particulate organic phosphorus, then more positive explanations might be offered for the variations that obtain. It is well-known that phosphorus assimilated by phytoplankton is again set free by processes of decay and digestion by other organisms. The precise location of this transformation and the rate at which it takes place are not clearly understood. Under certain conditions, phosphorus may be removed from active participation in the cycle or it may be conceived as existing predominantly in one particular form during specific intervals of time or space. Atkins (1930) and Cooper (1938) have attributed high phosphate concentrations to regeneration of phosphorus from decomposing floating organic material. The rapid utilization and regeneration of inorganic phosphorus observed in the Chesapeake waters has been interpreted on a basis of light available to the phytoplankton. The method of analysis employed is the variation of Denigès' ceruleomolybdate method described by Newcombe and Lang (1939).

RESULTS

The results of intermittent observations since 1936 on the distribution of nutrient salts in the Chesapeake Bay have clearly shown that these basic

* Contribution No. 32 of the Chesapeake Biological Laboratory.

substances do not follow any particular predescribed rule with respect to hourly, daily or seasonal changes. The physical and chemical properties of the Bay waters are essentially estuarine in character and, in the present

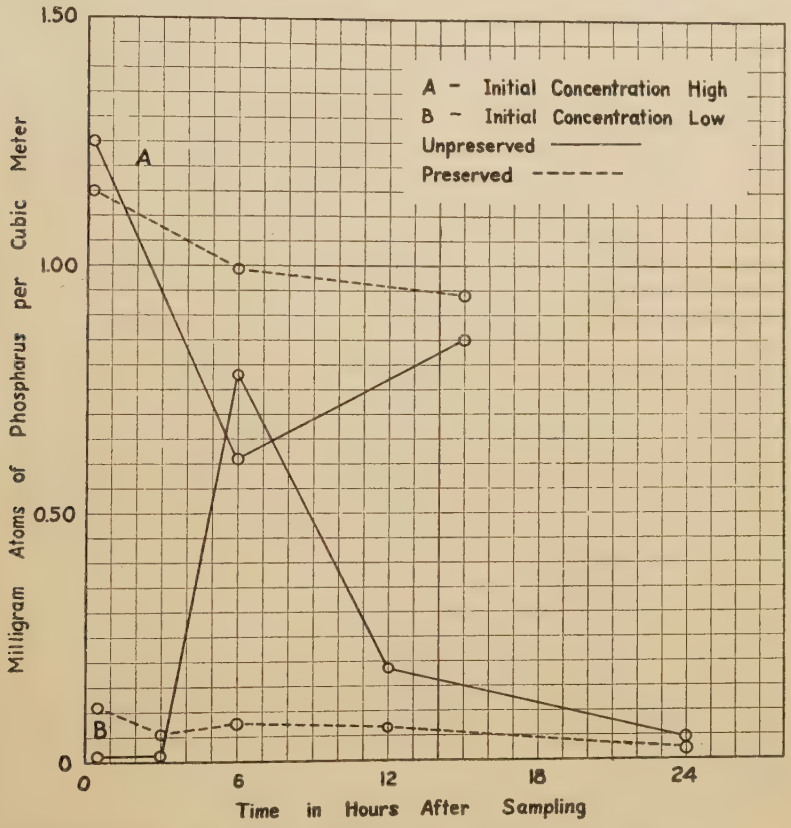

Figure 23. Showing the change in concentration of phosphorus accompanying storage with and without CS_2 preservative. – – – – – preserved; ———— unpreserved. Each point represents an average of two or three sample bottles. A, Initial concentration, high; B, Initial concentration, low.

state of knowledge, they are not particularly well understood. The characteristics treated here are: (1) effect of storage, (2) close interval changes with respect to space and time, (3) daily changes, (4) horizontal and vertical gradients.

Experiments designed to determine the stability of phosphate samples accompanying storage have shown that under the high summer tempera-

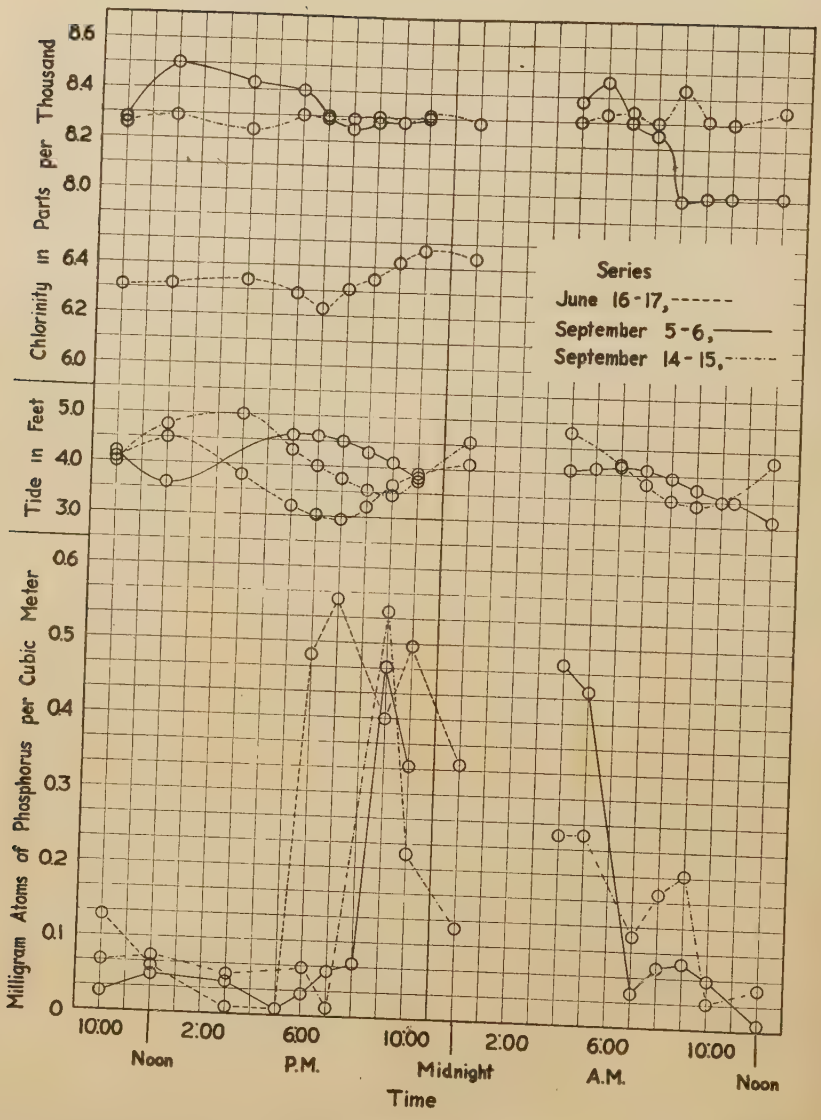

Figure 24. Variations at close time intervals of phosphate as mg. atoms P/m³ off the laboratory pier. – – – – – June 16–17; ———— Sept. 5–6; Sept. 14–15, 1939, -·-·-·-

ture conditions characteristic of the waters in this latitude, namely 25°–30° C., the phosphorus concentration may change significantly within a period of one hour. The character of this change is dependent, among other factors, on the light and temperature conditions during storage. In an effort to preserve samples so as to permit storage, the following preservatives were tested: chloroform, sodium fluoride and carbon disulphide.

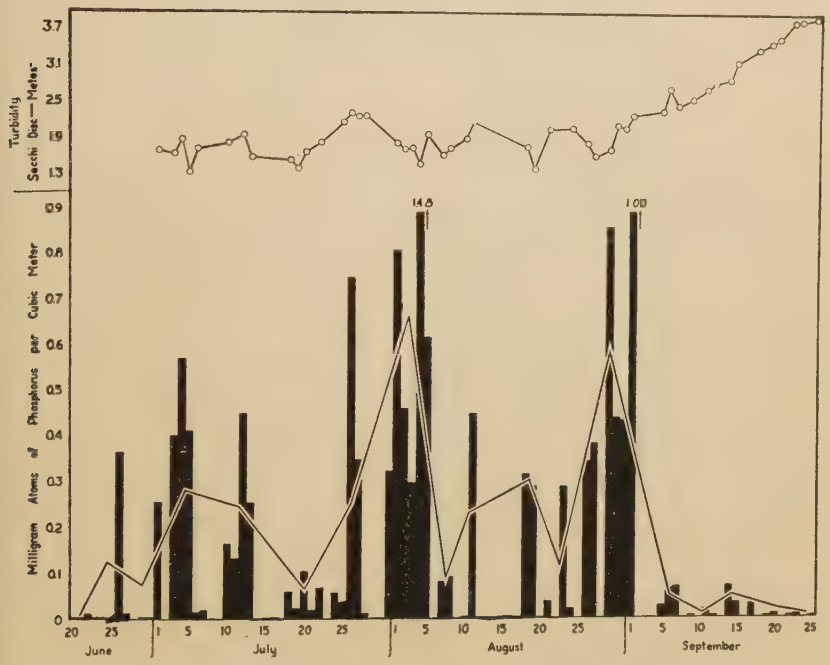

Figure 25. Daily concentrations of phosphorus at the end of the laboratory pier taken during period June 20–Sept. 25, 1939. Samples were collected in dark green bottles and analyzed immediately.

Chloroform and sodium fluoride are not satisfactory preservatives. Carbon disulphide has been tried frequently and in most instances significantly retards the rate of change that normally accompanies storage during the first fifteen hours. (Figure 23.)

Close Interval Changes.—The results obtained by Newcombe and Lang (1939) showing pronounced diurnal changes in concentration of phosphorus have been confirmed (Figure 24). During the periods of these experiments, the weather was clear and the wind movement light and moderate. The tidal amplitude and the accompanying chlorinity concentrations are presented in Figure 24. The results indicate a characteristic increase during

and following the hours of sunset and a corresponding decrease during the morning hours. Although considerable variations are known to exist between the different series that have been taken, nevertheless, there seems to be a general agreement with respect to the occurrence of higher concentrations during the hours of darkness. These findings are supported by random samples taken throughout the summer season.

Figure 26. Location of principal sampling stations (numbered points) in the Patuxent River (2 is equivalent to 102 and 108 is about 7 miles south of this station).

Day and night time samplings in the shallow waters between the end of the pier and shore have indicated that a decrease or increase may exist in the direction of the shore. These observations are important because of the possible significance they might be considered to possess in connection with the abrupt increase in concentration of P during the early hours of darkness. The changes are not considered to be greater than the normal variation obtained over this range of distance in the general region.

Daily Changes.—The results of daily analyses of water from off the laboratory pier during the period June 20–September 25 are shown in Figure 25. The highest values obtain during August with a slightly lower average during July. That a pronounced decrease takes place in September when turbidity is significantly reduced is clearly shown. This condition of

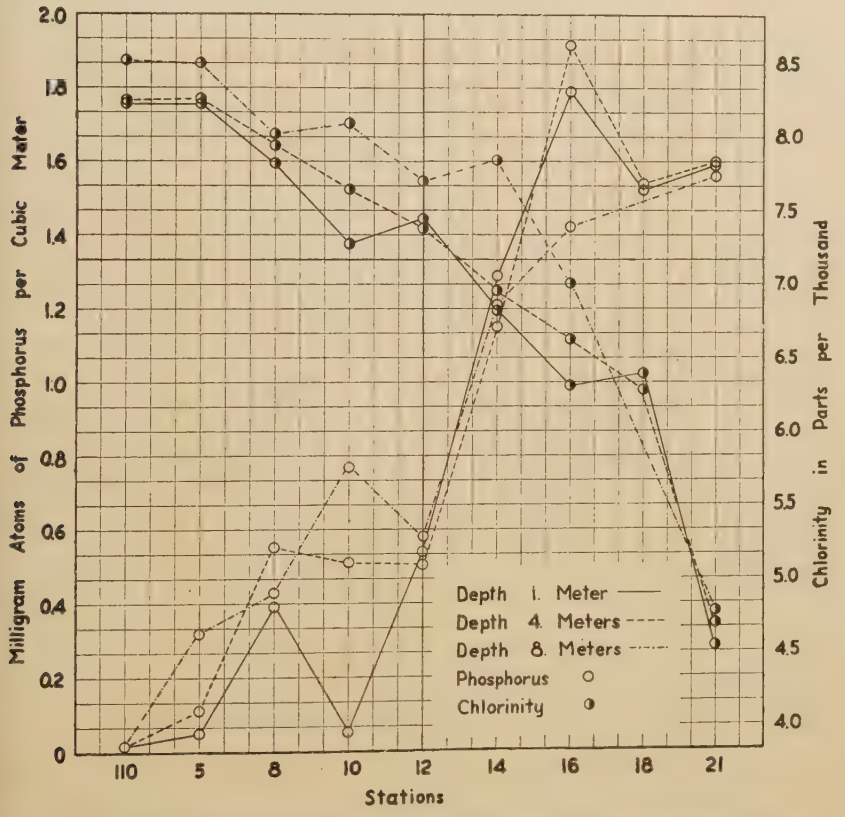

Figure 27. Horizontal gradients of phosphorus and chlorinity between stations 110 and 21. Time, Aug. 15, 1939, ————————, 1 meter, – – – – – –, 4 meters, –·–·–·–·–, 8 meters.

low phosphorus content and high transparency has continued during the fall period. The highest concentration recorded during summer is 1.48 mg. atoms per m.[3] It is not unusual to get mere traces in the pier samples, as is shown in Figure 25. The existence of marked daily variations in summer is quite evident.

Horizontal and Vertical Gradients.—Numerous horizontal sections taken during the different seasons of the year have indicated a conspicuous in-

TABLE I

VERTICAL AND HORIZONTAL GRADIENTS OF PHOSPHORUS AND CHLORINITY IN THE RIVER-BAY SECTION. TIME, AUGUST 15, 1939. PHOSPHORUS, MG. ATOMS PER M^3. CHLORINITY, PARTS PER THOUSAND BY WEIGHT

Depth in Meters

Station	*1*		*4*		*7*		*9*		*14*		*25*		*35*		*Tide*
	P.	*Chlor.*	*P.*	*Chlor.*	*P.*	*Chlor.*	*P.*	*Chlor.*	*P.*	*Chlor.*	*P.*	*Chlor.*	*P.*	*Chlor.*	
21	1.59	4.54	1.60	4.69	1.56	4.77									
18	1.52	6.39	1.54	6.27											
16	1.79	6.31	1.85	6.63	1.42	7.01									Ebb
14	1.29	6.82	1.15	6.96	1.21	7.84									
12	0.54	7.44	0.50	7.38	0.58	7.70									
10	0.05	7.27	0.51	7.64			0.67	8.09	0.60	8.48					
8	0.39	7.82	0.55	7.94			0.43	8.02	0.71	8.20	0.72	8.41	0.62	8.43	
5	0.05	8.22	0.11	8.26			0.32	8.50	0.38	9.44					
102A	Trace	7.99	Trace	8.01			0.54	11.81	0.73	12.30	1.11	12.96	1.65	13.22	Flood
110	Trace	8.23	Trace	8.25			Trace	8.52	Trace	11.09					

crease in phosphorus accompanying the decrease in chlorinity in the direction of Station 21 (Figure 26). The characteristic trend at the several levels studied is shown in Figure 27 and Table I. The horizontal gradients are subject to pronounced variations produced by mixing processes that may be influenced by tidal forces, wind movements, or excess drainage operating as single factors or in combination.

Table II shows the character of the horizontal gradient that may occur in the Bay waters during summer. This by no means represents a constant condition since hourly sampling in these waters has indicated that the changing position of water masses results in pronounced changes in the concentration of phosphorus at a given point (Figures 24 and 28). The table, therefore, simply represents normal expectancy for this region under prevailing chlorinity, temperature and light conditions.

The vertical gradients are conspicuously developed during the summer season. It is shown in Table I and Figure 27 that the subsurface levels may be expected to contain higher concentrations of phosphorus. Occasionally, low values have been obtained near the bottom, but usually there is evidence that this water had a recent origin near the surface. Concentrations above 1 mg. atom of *P* per cubic meter may be expected in the deeper waters whereas, in summer, but mere traces of phosphorus sometimes obtain in the surface level. The surface concentrations in the river waters are usually higher than those of the Bay and this is particularly evident following a period of rainfall.

DISCUSSION

Earlier studies at this Laboratory have provided considerable evidence of the existence of atypical phosphate conditions which seem to possess more than local interest (Newcombe, Horne and Shepherd, 1939; Newcombe and Lang, 1939). A continuation of these observations has brought to light certain basic facts that may not be universally recognized and has also contributed toward a more accurate picture of the variations in the concentration of phosphorus that characterize estuarine waters.

Little attention seems to have been given to the preservation of phosphate samples. When difficulty is experienced in running samples immediately on board boat, a definite problem arises in respect to the preservation of samples having a temperature around 26° C. at the time of collection. Ibanez (1933) favors the use of sodium fluoride as a satisfactory preservative. In several experiments, chloroform and sodium fluoride were employed without success. Carbon bisulfide reduces the change significantly for a short period and in past studies has been used (Figure 24). Recent experiments, however, strengthen the opinion that wherever possible phosphate samples should be run immediately, though collected in dark bottles and held in a cool place. Significant change may take place within

TABLE II

CHESAPEAKE BAY CROSS-SECTION AT STATION 108 SHOWING HORIZONTAL AND VERTICAL GRADIENTS OF PHOSPHORUS AND CHLORINITY. TIME, JULY 17, 1939. PHOSPHORUS, MG. ATOMS PER M.^{3} CHLORINITY, PARTS PER THOUSAND BY WEIGHT

	STATIONS								
Depth in	*West Shore*						*East Shore*		
Meters	*108a*	*108b*	*108c*	*108d*	*108e*	*108f*	*108g*	*108h*	*108i*
1 P	0.10	Trace	0.27	0.29	Trace	0.15	Trace	Trace	Trace
Chlor.	7.23	7.23	7.29	7.35	7.64	7.56	7.51	7.51	7.62
4 P		0.13	0.01	0.07	0.01	0.37	0.05	0.54	Trace
Chlor.		7.28	7.40	7.35	7.65	7.62	7.55	7.55	
9 P				0.06	0.14	0.15	0.20	Trace	
Chlor.				7.56	7.65	7.60	7.64	7.63	
14 P					0.21	0.23			
Chlor.					8.46	8.28			
19 P						0.08			
Chlor.						7.56			
25 P						0.53			
Chlor.						10.04			
30 *P*						0.57			
Chlor.						10.18			
Tide		Ebb						Slack	Flood

less than thirty minutes under the high summer temperatures of Chesapeake Bay waters.

The character of the close interval changes shown in Figure 24 is thought-provoking. These data confirm the general picture suggested by Newcombe and Lang (1939). The present state of knowledge with respect to phosphorus metabolism in the sea does not permit an explanation of this diurnal change. Available evidence does not point in the direction of the inflow of adjacent water masses as a possible explanation of the night and morning changes. There does not seem to be any conclusive evidence that would permit an explanation based on actual phosphate liberation by the living organisms in sufficient quantities to account for the observed fluctuations.

The pronounced diurnal variations suggests an absence or greatly reduced phosphate utilization during darkness and a marked liberation of phosphorus at this time. In fact, Renn (1937) discussing regeneration experiments states "if autolyzing bacterial cells liberate phosphorus as phosphate there always exists the possibility that diatoms and other phytoplankton may similarly bring about direct regeneration." Although his attempts to demonstrate such regeneration gave negative results, it was his opinion that the possibility definitely exists. Granting such an hypothesis, how can such a large phosphorus transfer be rationalized? If, in the presence of

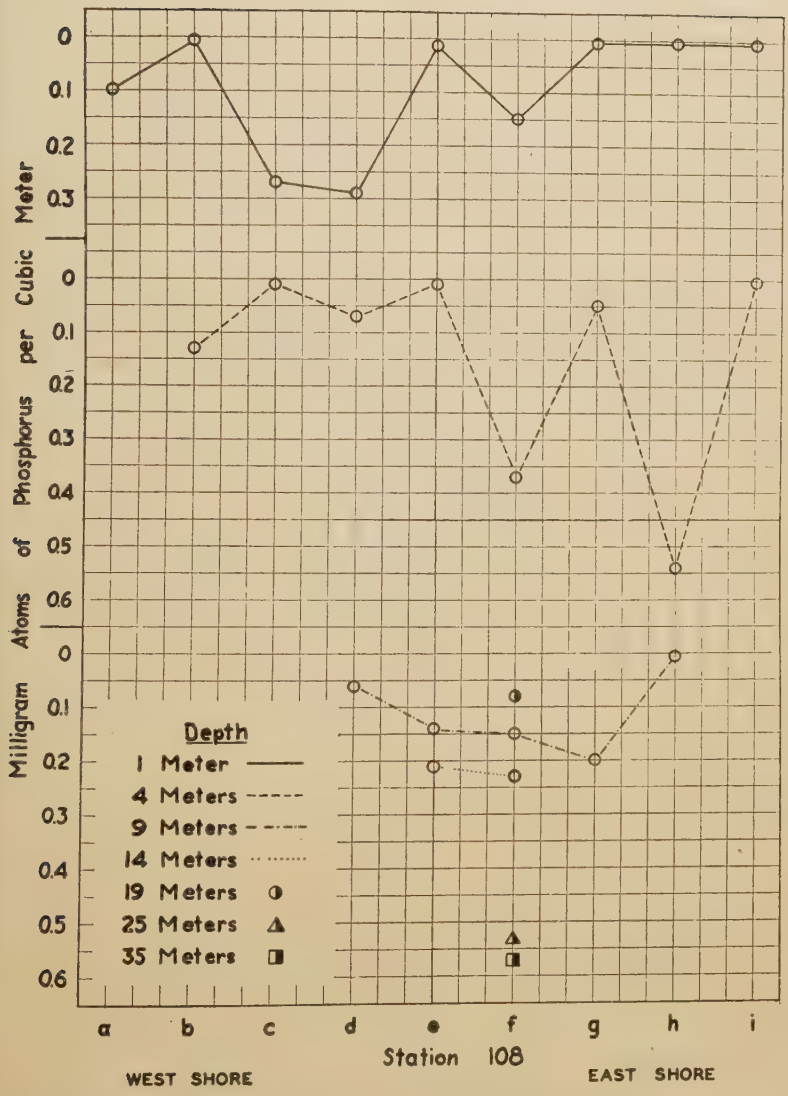

Figure 28. Chesapeake Bay cross section at Station 108 showing horizontal gradients of phosphorus at several depths. July 17, 1939.

an abundance of phosphorus, a diatom rapidly absorbed relatively large amounts of this element, it would seem reasonable to assume that a mechanism may exist within the cell whereby a speedy elimination of the phosphorus could be effected. Harvey (1937) has discussed the adsorption of phosphate on the surface of diatoms. He points out that colloidal and larger particles of phosphate can be utilized by and support the growth of diatoms. When more is known of the biochemical and biophysical processes involved in diatom metabolism, it may be possible to offer an explanation for the noticeably rapid utilization and regeneration of this important nutrient.

A conspicuous characteristic of the daily fluctuations indicated in Figure 25 is the sudden change that may occur within a short time interval. A careful examination of available data fails to offer an explanation. The highest average values are for August. Those determinations were all run immediately after collection on unpreserved samples. Some of those done during the summer of 1938 are on samples preserved by carbon disulphide which, it is believed, may account for the exceptionally high values obtained, on occasion, at that time. It is not uncommon to obtain concentrations of phosphorus above 0.2 mg. atoms per $m.^3$ in pier samples during late June, July and August. During the first part of September, 1939, an abrupt decline in concentration of phosphorus was observed in the waters off the laboratory pier. This change was accompanied by a marked reduction in turbidity as shown by Secchi disc readings. The increase in the transparency of the water is not explained by rainfall data nor by any other available observations. It is known that the condition of relatively low turbidity remains during the fall and winter seasons when the numbers of plankton organisms are greater. If an increase in phytoplankton occurs at this particular time when the stratum of photosynthetic activity is much deeper, then the decrease in inorganic phosphorus may be accounted for.

It seems desirable to point out the magnitude of the diurnal changes in concentration of phosphorus and the distinctive correspondence between its variations and those changes in submarine illumination referred to above. Results thus far seem to indicate that the light factor is of major importance in causing these variations as well as those horizontal differences that exist in the Chesapeake Bay waters (Newcombe and Lang, 1939). A conclusive answer to this problem must await information on (1) plankton abundance, (2) dissolved organic phosphorus, and (3) particulate organic phosphorus.

The horizontal sections of the Bay completed for the first time, show the normal expectancy in regard to stratification of chlorinity and phosphorus content (Table II). The highest concentrations in summer characterize the bottom waters whereas the smallest values are obtained in surface layers. Following periods of disturbance due to tidal action or winds or a combination of both, a fairly homogeneous condition is set up. It represents a

temporary dislocation and the stratified condition returns after varying time intervals depending on the particular condition involved. The chlorinity stratification perhaps returns first whereas there is a noticeable lag in the establishment of phosphate and oxygen levels of varying concentrations.

The horizontal changes in the Bay are considerable, as shown by the surface records in Table II (Figure 28). Horizontal variations accompanying decrease in chlorinity in an up-river direction are pronounced (Figure 27). In general, the phosphorus concentration increases at all depths accompanying an increase of river water. It has been suggested that the type of relationship represented in Figure 27 implies a lack of contribution of phosphorus to the Bay by the river waters and might be accounted for on a basis of utilization by river plankton according to the theory of Harvey (1928). Riley (1938) has presented very convincing evidence that the Mississippi makes substantial contributions to the waters of the Gulf of Mexico. In interpreting the condition shown to exist in the Patuxent River (Table I), further knowledge of the water movements and the results of the daily mixing processes and the subsequent dilution effects is highly desirable.

In general, the summer concentrations of phosphorus in the Patuxent River are above 1.0 mg. atoms per m.^{3} The effect of rainfall in increasing the river concentrations is immediate and quite significant.

SUMMARY

1. Experimental evidence is presented to show the need for immediate analysis of phosphate samples in order to assure best results for Chesapeake waters during summer.

2. Previous preliminary observations on the occurrence of pronounced diurnal variations in the concentration of phosphorus have been confirmed.

3. The daily variations in phosphorus content during the summer of 1939 were found to vary from mere traces to about 1.5 mg. atoms per m.^{3} Concentrations above 0.3 mg. atoms per m.^{3} frequently occurred. The peak seemed to have been reached in August and was followed by an abrupt decline in early September accompanying a marked reduction in turbidity of the local waters. This latter condition has continued during the fall period.

4. The waters of the Solomons Island region show a lack of homogeneity noticeably reflected in the horizontal variations in phosphorus content at the different levels. Highest concentrations occur in the bottom waters which frequently contain between 0.6 and 1.1 mg. atoms of P/m.^{3}

5. A pronounced increase in the concentration of phosphorus accompanies the decrease in chlorinity toward the headwaters of the Patuxent

River. The river waters usually range between 0.9 and 1.8 mg. atoms per $m.^3$ in summer.

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EDITORIAL NOTE

ON THE PREPARATION OF FIGURES FOR PUBLICATION

Experience in the publication of the first two volumes of this Journal indicates a considerable amount of uncertainty among authors and illustrators in the choice of lettering-sizes suitable for a clearly readable reproduction. The accompanying diagram (Figure 29) has therefore been prepared as a guide for determining height of letters and size of symbols to be used on original drawings.

The three curves indicate the height in *millimeters* (scale at right) of Large, Medium and *Smallest permissible* letters suitable for use on *original drawings* according to the amount of reduction (scale below) for which they are intended or which they require for publication. Rates of reduction between 1 : 3 and 1 : 6 are given on a smaller scale than that used for those between 1 : 1 and 1 : 3. The words (LARGE, MEDIUM, SMALLEST) used at the left to identify each of the 3 curves also illustrate the appearance *after reduction* of original lettering selected by the use of this diagram.

Authors wishing to use special dimensions for final reproduction should compute the rate of reduction involved and use the scale at the bottom. Those who merely wish to use the maximum size available on the pages of this Journal may take advantage of either of the scales given at the top, by which the usable heights of lettering may be determined directly from the overall width of the original, measured either in centimeters or in inches. The printed page of this Journal measures 6½ by 4¼ inches, or 16.5 by 10.8 centimeters. For figures intended to be viewed with the page held in normal reading position the maximum width is thus 4¼ inches and the maximum depth 6 inches or 15.2 cm. (to allow for legend) and the lower scale given above the diagram in Figure 29 should be used. Illustrations demanding a greater width may be arranged to be viewed with the page held at right angle to normal reading position, as done with Figures 16 and 17 on pp. 66 and 68 preceding. In these cases the upper scale above the diagram should be used for determining letter size, and the maximum permissible height of the illustration after reduction is 4 inches or 10.2 cm.

Attempts to reproduce an illustration without reduction normally results in a coarsening of details. *A minimum reduction at the rate of* 1 : 1.2 *is desirable for clearness,* and this is the least reduction which will be applied to any drawing submitted for use in this Journal.

It is desirable to limit all lettering to *capital letters only.* Lower case letters of corresponding size are less readable after reduction. *If mixed capital and lower case characters are used, the letter-height suggested in Figure 29 as "Medium" (for capital letters alone) should be regarded as minimum size in the mixed composition,*

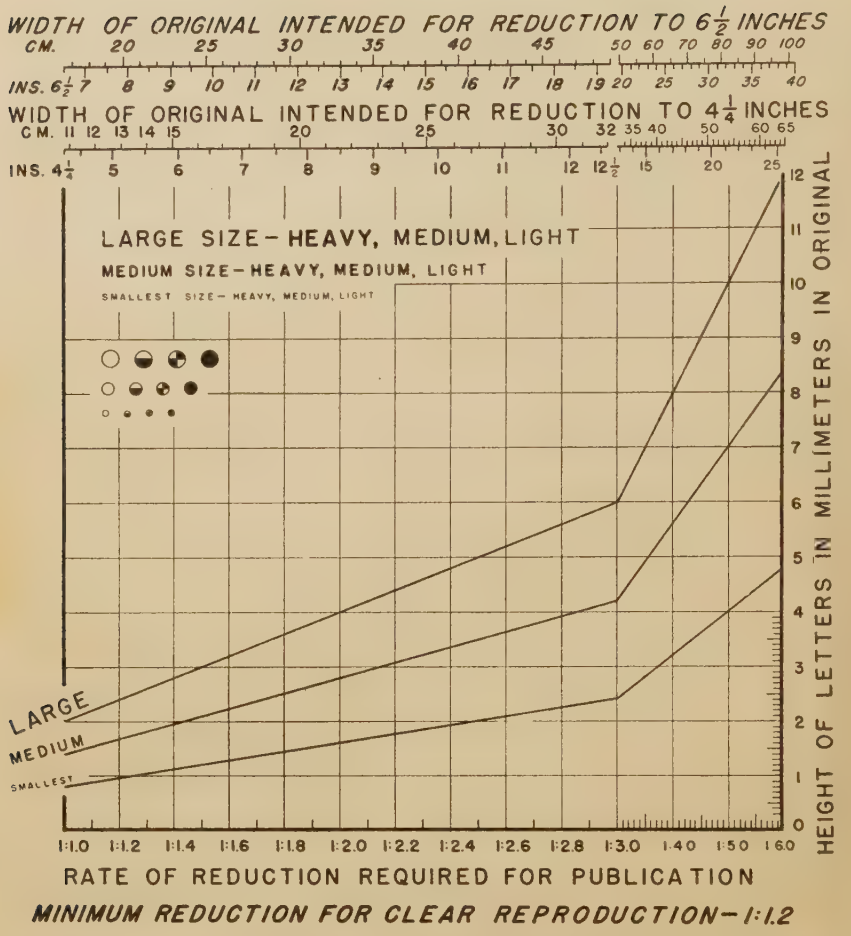
WIDTH OF ORIGINAL INTENDED FOR REDUCTION TO 6½ INCHES
CM. 20 25 30 35 40 45 50 60 70 80 90 100
INS. 6½ 7 8 9 10 11 12 13 14 15 16 17 18 19 20 25 30 35 40
WIDTH OF ORIGINAL INTENDED FOR REDUCTION TO 4¼ INCHES
CM. 11 12 13 14 15 20 25 30 32 35 40 50 60 65
INS. 4¼ 5 6 7 8 9 10 11 12 12½ 15 20 25
LARGE SIZE – HEAVY, MEDIUM, LIGHT
MEDIUM SIZE – HEAVY, MEDIUM, LIGHT
SMALLEST SIZE – HEAVY, MEDIUM, LIGHT
LARGE
MEDIUM
SMALLEST
HEIGHT OF LETTERS IN MILLIMETERS IN ORIGINAL
0 1 2 3 4 5 6 7 8 9 10 11 12
1:1.0 1:1.2 1:1.4 1:1.6 1:1.8 1:2.0 1:2.2 1:2.4 1:2.6 1:2.8 1:3.0 1:4.0 1:5.0 1:6.0
RATE OF REDUCTION REQUIRED FOR PUBLICATION
MINIMUM REDUCTION FOR CLEAR REPRODUCTION – 1:1.2

Figure 29. Curves for the determination of capital letter heights for use in original drawings. See text.

LARGE SIZE – HEAVY, MEDIUM, LIGHT

MEDIUM SIZE – HEAVY, MEDIUM, LIGHT

SMALLEST SIZE – HEAVY, MEDIUM, LIGHT

Figure 30. Appearance of lettering from figure 29 *before* reduction at the rate of 1 : 1.6

It should be noted that slanting letters appear larger than straight letters of equal size. The captions for the two scales above the diagram in Figure 29, starting with the words "WIDTH OF," are lettered with characters of the same height and weight of line to illustrate this point.

Samples of heavy, medium, and light lettering in Large, Medium and Smallest permissible size are included in the diagram. This sample legend is also reproduced separate in Figure 30 as it would appear in an original intended for reduction at the rate of 1 : 1.6.

The curves for suitable letter-heights also give the usable transverse dimensions of identifying marks such as circles and dots as shown by the examples entered in the figure. The following reservations apply to the use of the curves to determine the required size of identifying marks in the original. 1) A diameter corresponding to the smallest permissible height of capital letters *can only be used when not more than 2 kinds of identifying marks* (circles and dots) *are needed for differentiation.* If it is necessary to differentiate between *3 or more* identifying marks the *smallest* usable diameter should correspond to the *medium* height of capital letters. 2) *Polygonal forms should be avoided* so far as possible. If absolutely required to obtain sufficient variety of identifications, one polygonal form (triangles) may be used when dimensions not smaller than the medium height of capital letters are employed. For 2 polygonal forms (triangles and squares) a size of identifying marks not smaller than the height of "LARGE" capital letters is desirable. More than two types of polygonal forms* (in addition to circular marks) should not be used under any circumstance. Crosses, or crosses enclosed in circles are less desirable than the types of circular marks suggested in Figure 29, and *require a larger minimum size.* Whether or not they are preferable to polygonal forms when additional differentiating marks are absolutely required is a matter of taste.

The use of cross-section paper ruled or printed with *blue* lines can not be too strongly recommended. These lines disappear in the process of reproduction without any necessity of retracing the drawing or using color-filter methods, at the same time as all the advantages of the cross-rulings are preserved for the checking of figures with tables, etc., in the writing and editing of manuscripts. Figure 29 has been drawn on blue cross-section paper of which only the lines emphasized in black India ink remain in the reproduction.

A. E. PARR D. T. PITCHER

* Each of which may be used in both black and white (outline) form on the same graph.

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